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UPPER WIND STRUCTURE

BY

T. O. ERIKSSON

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UPPER WIND STRUCTURE

INAUGURAL DISSERTATION

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UPPSALA 1961

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CONTENTS

Page

FOREWORD 5

ABSTRACT 6

INTRODUCTION 7

Part 1. VARIABILITY OF UPPER WIND WITH TIME

1. Some previous investigations of upper-wind variations 9
2. Some points of view concerning wind variations and wind variability . . . 13
3. Material and methods 17
   - 3.1 The measurements 17
   - 3.2 Some theoretical considerations 18
   - 3.3 Treatment of obtained wind values 24
   - 3.4 The standard vector deviation as an expression of the wind variability . . . 27
   - 3.5 Errors 31
4. Wind-variability values obtained 36
   - 4.1 Meteorological conditions during some serial pilot-balloon ascents 36
   - 4.2 Discussion of obtained values 38
   - 4.3 Influence on the wind variability from altitude, synoptic situation and wind speed 44
5. A connection between wind variability and time interval 46
6. Some applications of the wind variability 52

Part 2. UPPER-WIND STRUCTURE IN FRONTAL LAYERS AND INVERSIONS

1. Introduction 55
2. Visualization of the wind conditions 56
3. Examples 58
   - 3.1 Warm front and back-bent occlusion 58
   - 3.2 Cold front 66
   - 3.3 Occlusion front 75
   - 3.4 Inversion layers 78
4. Conclusions 81

REFERENCES 84

FOREWORD

The present study was initiated by the Swedish Artillery Council. It has been carried out with the assistance of many institutions, which I have the pleasurable duty of thanking. Preliminary results were rendered in a memorandum from the Headquarters of the Swedish Military Weather Service in the spring of 1958. A more complete presentation was given in a report from the Research Institute of National Defence in May 1959.

The basic data are measurements of the wind aloft made during several periods during the years 1953–1957. These data were kindly put at my disposal by the Swedish Anti-Aircraft Artillery. Most of the measurements were made by the Swedish Anti-Aircraft Artillery Firing School. I am especially indebted to Colonel B. Olin, Lieutenant-Colonel E. Sundin and Major K. Karlsson.

Measurements made at the aerological station at Bromma were also used in this work. The original data from a number of these measurements were kindly put at my disposal by the Meteorological and Hydrological Institute of Sweden.

The processing of the observational material, like the analytical and statistical work, was begun at the Headquarters of the Swedish Military Weather Service. For all the assistance rendered me I wish to express my sincere thanks to Colonel O. Herrlin, head of the Service.

The main part of the study was carried out at the Research Institute of National Defence. For the resources the Institute placed at my disposal and for technical help received I wish to express my sincere thanks to Dr. T. Magnusson, head of FOA 4. I am also indebted to colleagues for their great interest showed in discussing certain parts of the work.

I am greatly indebted to Professor G. Liljequist and Professor E. Palmén, who from time to time during the course of the study gave me very valuable advice and showed an interest in the work which has been most stimulating to me. I also wish to express my sincere thanks to Professor T. Bergeron for his interest in discussing some of the vertical cross-sections in this study.

Finally, I would like to thank Mrs. L. Edenberg for her efficient help in typing the manuscript.

Stockholm, February 1961

T. O. Eriksson

ABSTRACT

Data from 98 serial radar measurements of the upper winds between 1 and about 10 km and from the years 1953–1957 were used to investigate the variability of upper winds with time, i.e. the wind variations with time in a quasi-stationary wind field. The wind-variability values given are corrected for instrumental errors. Information of the magnitude of the variability is of value in experimental tests within the free atmosphere. Decisions on what length of time an upper-wind measurement is representative of the real wind conditions, how accurate instruments for upper-wind measurements should be, and so on, depend among other things also on the variability. The relation between variability and synoptic situation, wind speed and altitude was studied.

Concerning the synoptic situation, variability values were found to be distinctly lower in ridges than in troughs when the wind speed was lower than 5 m/sec. At higher wind speed there was no difference between the variability in ridges, troughs and situations with straight contour lines.

The variability for a given time interval was found to increase with increasing wind speed. This increase, however, ceased when the wind speed had reached about 15 m/sec.

The altitude as such had no influence on the variability from the top of the frictional layer and up to about 9–10 km. In the stratosphere, however, there were found significantly lower variability values than in the troposphere. Since on an average the wind speed increases with altitude to a level about 7–9 km the altitude can indirectly influence the variability.

Some examples are also given in order to illustrate the upper-wind conditions in connection with frontal layers. It was shown that the streamlines may have an anticyclonic curvature behind a cold front not only at levels near the earth's surface, but also at levels up to 700 mb. A wind-speed minimum within frontal layers seems to be a characteristic feature of the wind structure in connection with fronts.

INTRODUCTION

The use and importance of upper-air wind data continue to increase. One reason is the great demands which are placed on accuracy and on the possibility of control in activities such as artillery firings, flight planning and flight testing of modern high-performance aircraft and research into missile operations, the behaviour of explosion debris clouds, etc. The demands on accuracy of the information of the wind required by these activities also continue to increase and often they exceed the accuracy otherwise required in applied meteorology.

Mere averages in time or space—the usual information on the wind in both forecasts and wind reports—are not always satisfactory in activities such as those mentioned above. It is often necessary to have exact information of the wind conditions at a given moment. The usual method of determining the winds aloft, however, does not permit simultaneous measurement and use of the wind data. The time interval between measurement and utilization will generally be one or two hours. Since the wind at any level will not remain exactly constant during this interval, this gives rise to an error in the data presented to the consumer. This error can be taken into consideration by using a wind-variation parameter. Such a parameter, however, depends on the synoptic situation. It has a larger value in a fast variable situation than in a situation with only small changes. The present study treats with wind variations in the latter type of situations. These variations are here called the *variability* of the wind.

Wind information cannot always be obtained in a direct way by measurements. Therefore geostrophic wind values often must be measured on contour charts or forecast contour charts for selected isobaric surfaces, extrapolating and interpolating as necessary. By this method a number of errors occur which influence the wind values presented to the consumer. Especially within areas where fronts occur it is a common experience that the difference often is large between the real wind and the wind values obtained from a contour chart. In part, this difference is due to the general tendency of drawing contour lines with a cyclonic curvature through fronts. Probably this method of drawing is due to the fact that our knowledge of the wind conditions in connection with fronts is obtained in an indirect way, *inter alia* by means of the thermal wind formula. Therefore the normal contour lines do not always exactly describe the actual wind conditions in areas where fronts occur.

A thorough study of the conditions mentioned above would require a large number of upper-wind measurements made at short time intervals under different weather conditions and during every season. It would also be desirable to have measurements

from several places distributed over the different parts of the country. To make such a large number of measurements is, however, very expensive and therefore it has been possible to make only a small number of measurements for the special purpose of such a study. Data from measurements of winds aloft made for other purposes—and not as appropriate as desirable—have therefore also been used.

The data obtained have been analyzed in the present study. In Part 1 the variability of upper wind with time is treated, and in Part 2 the structure of upper winds in connection with frontal and inversion layers.

Part 1

VARIABILITY OF UPPER WIND WITH TIME

1. Some previous investigations of upper-wind variations

The importance of the variation of upper winds was first noted by ballistic meteorologists during World War I, when artillery had reached such a development that the disturbances caused by the wind in the projectile trajectory could be determined and separated from other disturbances and when the pilot-balloon method of measuring winds aloft had become common. The problem was probably first dealt with by Durward (1921) on the basis of pilot-balloon measurements made on the western front during World War I. However, no results of this investigation have been accessible to the present author. On the whole, only a few and inextensive investigations of the variation of upper winds seem to have been carried out before World War II. Since then, however, after electronic equipment for measuring upper winds had become common, the activity has grown more extensive. Apart from the new measuring methods, this increase in activity is also due to an increasing interest in the variations of winds aloft.

The basis for the investigations is either measurements for the daily weather service or for special purposes, in which latter case the method of serial ascents has normally been used. Here the balloons have been released with fixed time intervals, varying in length from investigation to investigation. Generally the variations are studied as a function of time, and only a few investigations also study the variations as a function of the distance between the observation places. The investigators generally use different methods in their analyses and the results are given in different ways. This makes it difficult to give a summary of and a comparison between the results, but in spite of this an attempt has been made to do so.

Many papers have been written on upper-wind variations and all cannot be reviewed here. The abstracts given are short extracts from articles in the meteorological literature; consequently they cannot be complete. The abstracts are only from those parts of the articles cited which deal with wind variations with time. English units of measure used in the original works are converted into metric ones and the results are rounded off.

Some investigations describe the variations that occur during a single day and others give a statistical treatment of variation measurements from periods of varying length. The former usually describe the variations with the help of isopleth diagrams,

a method which gives a very clear picture of the variations during the measuring period but does not permit any direct, quantitative comparison between the variations on different occasions. Moreover, there is a difficulty with this method. Direction and speed must be visualized separately, and so it is hard to get an idea of the simultaneous variation of direction and speed.

The method has been used by Mildner *et al.* (1931), by Mildner (1933) and by Loeser (1942). All of these investigations describe results from serial ascents made during single days. The isopleth diagrams of Mildner (1931) describe the variations within the 0–1800 m layer during a period of 8 hours on October 15, 1930. The diagrams of Mildner (1933) deal with the 0–700 m layer during 7 hours on October 20, 1931. The diagrams of Loeser describe the variations on two different days (November 24, 1938, and November 22, 1939), the former within the 0–2200 m layer, the latter within the 0–2800 m layer, and both during 4 hours.

No general conclusions can be drawn from these results. However, in order to obtain a comparison with the other investigations from which abstracts are given here, wind-variation values for certain time invervals have been estimated from the isopleth diagrams. As the other investigations mentioned below dealt with the conditions in the free atmosphere, variations are only estimated from the investigations by Mildner (1931) and Loeser and for the level of 1600 m, the highest suitable joint level of the diagrams. As only a few values can be obtained from the diagrams, our estimation is very rough and must be regarded only as a sample. In Fig. 1 these estimates are represented together with wind-variation values from other investigations mentioned below. The wind-variation values are given in m/sec as a function of time.

The wind-variation values obtained from Mildner (1931) are represented by the straight line 1 in Fig. 1. The wind speed was in this case about 3 m/sec. From the two cases treated by Loeser lines 2a and 2b are obtained. The wind speed was in the former case 9–15 m/sec and in the latter about 2 m/sec.

In those investigations where the data obtained were numerous enough to permit a statistical treatment, the authors have calculated the vector difference between wind values obtained for certain time intervals. The result has then been given in the form of a statistical parameter, usually the standard vector deviation, the median or the probable vector deviation. In the cases where median or probable vector deviation values were given, the present author has transformed them to standard vector deviation (under the assumption that the individual vector differences form a normal circular vector distribution) by the following expression (see, for instance, Brooks & Carruthers, 1953, p. 196):

$$\frac{\text{median}}{0.83} = \frac{\text{probable vector deviation}}{0.83} = \text{standard vector deviation.}$$

In order to perform this calculation, the data given in the works under consideration must often be taken from indistinct diagrams and therefore the curves shown in

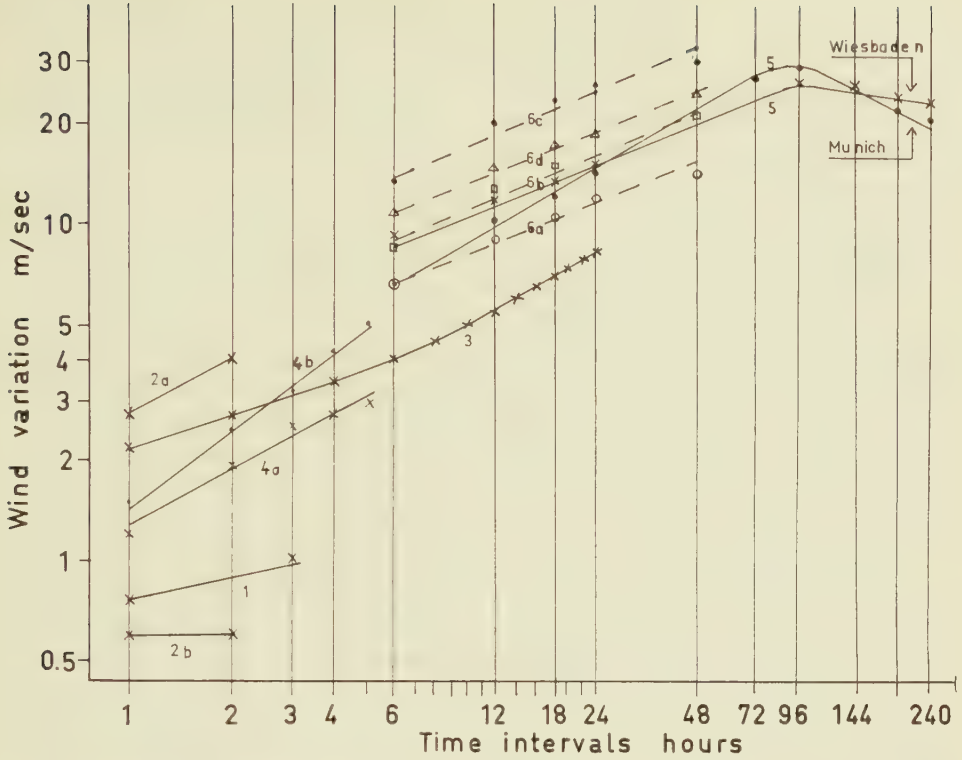

FIG. 1. Diagrammatic representation of wind-variation values from different sources. 1: Mildner (1931), 1.6 km, wind speed 3 m/sec. 2: Loesner (1942), *a*. 1.6 km, wind speed 9–15 m/sec; *b*. 1.6 km, wind speed 2 m/sec. 3: Anon. (1940), 0–4.6 km. 4: Singer (1956), *a*. 3–3.5 km; *b*. 9.1–11.6 km. 5: Smith & Wright (1957), 0–15 km. 6: Durst (1954), *a*. 3.0 km; *b*. 5.6 km, *c*. 9.5 km; *d*. 11.8 km.

Fig. 1 may not be entirely correct. However, this does not affect the discussion in section 2.

One author (Anon., 1940) has from measurements of shell-bursts, smoke puffs and from double-theodolite observations of pilot balloons investigated the wind variations up to a height of 4600 m. *The constancy of the wind variation with height is pointed out as a noteworthy result.* Standard vector deviations are estimated from a curve in the investigation, giving probable vector wind change for time intervals from 1 to 24 hours, which is considered to have application to all heights up to 4600 m. The curve given is corrected for probable errors. The standard vector deviations estimated from this curve are shown by line 3 in Fig. 1.

Singer (1956) has made use of 226 triple-theodolite observations of pilot balloons up to a height of 11,600 m for the period July 5, 1949–June 28, 1950. Singer examined separately the four layers 900–2700, 3000–3500, 6100–8500 and 9100–11,600 m. He

established by means of first quartile, median and third quartile of the magnitude of wind vector differences obtained *that the wind variation is greater in the lowest layer than in the layer above it and that the variation increases with height from the second layer upward.* The result is shown in diagrams from which the standard vector deviation is estimated for the layers 3000–3500 and 9100–11,600 m. The result appears from lines 4a and 4b in Fig. 1.

Smith & Wright (1957) used the synoptical pilot-balloon measurements from Munich, Wiesbaden, Hanover and Berlin during the period November 20 through December 13, 1956 in order to calculate for each place separately a mean vector for the whole 0–15,000 m layer. These vectors form the basic data from which the wind variation was calculated. Standard vector deviations indicated by Smith & Wright for Munich and Wiesbaden (the places with most data) are shown in curves 5 in Fig. 1.

Gabriel & Bellucci (1951) used 94 pilot-balloon measurements from a place in New Jersey from the period August 1943–March 1945 in order to investigate the wind variation as a function of height. The time difference between the measurements varied from 50 minutes to 2 hours. The authors calculated the median value of the wind vector differences for every 900 m in the 1500–8500 m layer and found by means of Student's *t*-test that there was no significant difference between the values for each separate height and the mean value of all the heights together. The authors therefore regarded it as confirmed *that the wind variation does not vary with height in the layer examined, 1500–8500 m.*

Durst (1954) showed in a large investigation how the variation of upper winds with time can be estimated for every place on earth, if certain coefficients in a common expression for the variation are known. The expression given by Durst is

$$\sigma_t = \sigma \sqrt{2\,(1 - r_t)},$$

where σ_t is the root-mean-square difference between wind at one time and t hours later;

r_t is a correlation coefficient; and

σ is the standard vector deviation of the winds considered, i.e. a climatological parameter obtained from many years of upper-wind measurements.

Durst now made an examination of the variation of wind at Larkhill for the period July 1946–June 1947 and thus determined the value of σ_t for the time intervals 6, 12, 18, 24 and 48 hours. By using the expression above, it was then possible to calculate the values of r_t after computing σ, which was done from upper-wind measurements also made at Larkhill during the period November 1939–May 1945. It was found that r_t decreased with the length of the time interval but was practically constant for all heights. Winds from other places all over the world were also investigated and r_t was found to be practically constant for all extra-tropical places. Thus by using the values of r_t calculated by Durst, and after calculating the value of

σ from upper-wind measurements for several years from an actual place, it is possible to substitute values of t in the expression above and obtain the variation to be expected.

As an example of σ_t for various values of t lines 6a–d in Fig. 1 are drawn. They are based on values from Larkhill for the period July 1946–June 1947.

2. Some points of view concerning wind variations and wind variability

In the investigations mentioned above, practically nothing emerges about how the wind variations observed are affected by meteorological conditions such as the synoptic situation or the wind speed. Only the influence of height is discussed and two opinions appear: one that the height has no influence on the wind variations (Anon., 1940; Gabriel & Bellucci, 1951) and another that such an influence does occur (Singer, 1956).

A glance at Fig. 1 seems to show that the wind-variation values increase with height. But this influence may be indirect and caused instead by the variation of the wind speed with height. Since, judging from measurements over the southern part of Sweden, the wind speed on an average increases with height to a maximum of about 7–9 km and above that level decreases with height, it seems an acceptable assumption that the wind speed alone can influence the variation of wind in the way mentioned. This assumption is supported by lines 6 in Fig. 1, in which the wind variation increases from 3000 m (line 6a) to 9500 m (line 6c) and then decreases from 9500 to 11,800 m (line 6d), i.e. about the same variation with height as the wind speed itself normally has. The result from the single days shown by lines 1, 2a and 2b also seems to verify the influence of wind speed, as this was low in 1 and 2b with small wind variations, but high in 2a with large wind variations. The difference between curve 3 and curve 6a (3000 m) during time intervals between 6 and 24 hours can also be explained by differences in the wind speed in the two cases. Curve 3 is obtained by values from the whole 0–4600 m layer and by optical measurements, and therefore the low heights seem to be over-represented, whilst curve 6a is obtained with electronic equipment and based only on observations from the height of 3000 m. It is therefore probable to assume that, on an average, the wind speed was greater in case 6a than in case 3. The same reason may explain the difference between curves 3 and 5 and also between lines 4a and 4b by Singer, who maintained that the variation increased with height as such.

One can also conceive that the difference between the curves is caused by meteorological conditions other than the wind speed. It is well known that the wind is gusty in cold air masses but not in warm air masses. However, most of the investigations comprise such long periods that practically all weather conditions ought to have occurred in all of them. Consequently, meteorological conditions other than the wind

speed do not seem to be responsible for the differences between the curves. However, this does not mean that meteorological conditions do not influence the wind variations.

Of all the investigations mentioned above, only Durst's work has yielded a result which can directly be used generally. His method is, however, based on the climatological variation σ, which must be known or be computable from pilot-balloon ascents taken over a long period. Existing series of pilot-balloon ascents, however, often originate from optical single-theodolite measurements and are therefore misleading, because data are obtained only when the balloon is not hidden by clouds. Also strong winds are under-representated, owing to the disappearance of the balloon in the distance. The method cannot be used for places without a long period of upper-wind measurements. In connection with this it seems suitable to mention that in a work by Brooks *et al.* (1950) values of σ are mapped for the whole world. These maps can substitute the computing of σ but, even for the maps, what is said above about existing series of pilot-balloon ascents is valid. It is also worth nothing that the σ_t of Durst is not the same as the variability in that sense in which the word is used in this investigation (see below). The σ_t of Durst is a function of σ, but σ is calculated from many years of upper-wind measurements without any regard to the synoptic situation and its variations. Therefore σ_t is rather a climatological element while the variability used in this study is a factor which can be fitted to actual meteorological conditions.

The works reviewed above *do not sufficiently take account of the instrumental errors.* In some cases the authors state that the errors are very small, but only in one investigation (Anon., 1940) the errors are separated from the wind-variation values obtained. *Such a separation seems to me, however, to be essential.*

The wind variations constitute a wide spectrum. The small-scale turbulence with periods from a fraction of a second to several minutes belongs to one end of this spectrum. Because of its importance in atmospheric pollution and for military operations, the small-scale turbulence has been studied intensively and often with very satisfactory results. These small-scale fluctuations, however, appear to be fairly random. This means that their combined effect on ballistic or synoptic-scale forecasting problems will normally cancel out and can thus be neglected in such problems with a good degree of confidence. To the other end of the wind-variation spectrum belong those variations which are due to the movements and changes in intensity of cyclones and anticyclones and phenomena associated with these, i.e. having periods of a day or two. These variations belong to the problems of weather forecasting. They are fairly well characterized by the different prognostical methods which are in use. Between these two outer limits there are, however, variations which have been investigated only to an insignificant extent—at least judging from the literature. These variations of the wind could be defined as the vector difference between two wind values obtained by two measurements performed at the same place, but at

different times; both, however, in a synoptic situation when no changes on a large scale can be observed from the meteorological maps.

The wind-variation values which are given in the investigations mentioned above must in part be ascribed to variations belonging to the middle part of the spectrum. They can, however—and this is very likely—also have been influenced by wind variations due to changes in the synoptic situation. In any case, nothing indicates that the authors have tried to separate the two kinds of wind variations, i.e. those which are due to changes in the synoptic situation and those belonging to the middle part of the wind-variation spectrum.

Sometimes it is important for some special purpose that the wind variations should be as small as possible. It is therefore often necessary to choose a synoptic situation when the wind variations can be supposed small. Wind variations, however, *always* occur and therefore it is necessary to take them into consideration. This can be done by using a wind-variation parameter. If such a parameter has been obtained from measurements independent of whether the weather situation has been stable or unstable with fast-moving cyclones or troughs, it is unnecessarily large if used during a rather stable synoptic situation. Even a wind-variation parameter calculated for a time interval as short as one hour may be considerably influenced, if the synoptic situation changes rapidly.

In upper-wind forecasts the changes of the synoptic situation are taken into account. But the prognostic wind value given to a consumer is often found to differ from the actual wind. This is often the case even if the changes of the synoptic situation during the range of the forecast are correctly predicted. Then the difference must be due to variations belonging to the middle part of the wind-variation spectrum.

In order to investigate this type of wind variations the present study deals with wind-variation values calculated from serial pilot-balloon ascents made during situations when the wind field—as it appears on the upper-air maps—was quasi-stationary during the measuring period. These wind variations are here called the *variability* of the wind.[1] Knowledge of the variability thus makes it possible to estimate those variations of the upper winds which are inaccessible by synoptic-scale observation technic and synoptic-scale forecasting.

Of course, no distinct difference exists between the *variability* of the wind and the small-scale end of the wind-variation spectrum on the one hand, and the large-scale end on the other. Therefore some parts of the variability may depend on small-scale turbulence and some parts of large-scale motions. But in spite of this it seems suitable to use the conception of *variability* of the wind instead of the wide conception of variation or of turbulence, as the following example will show.

In one investigation of predicting radioactive fallout it is stated that "Turbulent diffusion—in at least one of the models which has been tried the spread of the

[1] Several authors use the word variability, not in the sense given here, but merely as another word for variation.

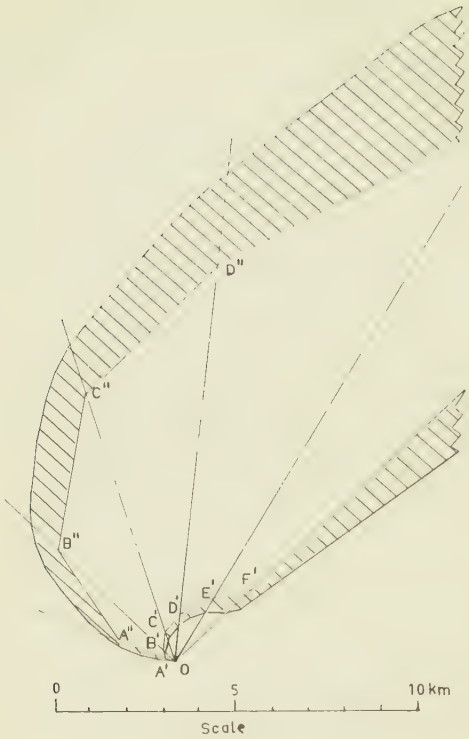

FIG. 2. Example of a fallout plot. (From *Meteorology and Atomic Energy*, Washington, 1955.)

trajectories due to random turbulence has been taken into account. However, most of the models choose to neglect this effect, since it does not appear to be very important for the early deposition".[1] In another investigation quite the contrary is stated: "To make an approximate correction for small-scale diffusion, displace points A′B′C′, and A″B″C″ (Fig. 2), so as to make for the maximum increase in the fallout area, about 1 mile for each 7 miles distance from the burst site."[2] The burst site is at 0 in Fig. 2 and the method mentioned gives a quite considerable enlargement of the fallout area, as is to be seen from the hatched area in Fig. 2. The former investigation probably considers small-scale turbulence and the latter rather the spread caused by the *variability* of the wind.

From *Meteorological Abstracts and Bibliography*, published by the American Meteorological Society, it appears in the numbers from January 1958 and May 1959 that a number of investigations into the variation of the upper wind with time have been carried out in the U.S. during recent years. A summary and analysis of data

[1] *The Nature of Radioactive Fallout and its Effects on Man.* Part 1, p. 109. United States Government Printing Office, Washington, 1957.

[2] *Meteorology and Atomic Energy*, p. 79. Prepared by United States Department of Commerce, Weather Bureau, Washington, 1955.

obtained during previous projects was given by Arnold & Bellucci (1957). A report by Plagge & Smith (1956) treats the result obtained during Project Rawijet, a study of the wind variations in space and time at the Salton Sea Test Base. From the abstracts given, it does not seem probable that the variability, in the sense used in the present study, has been treated. According to the abstracts, the following factors influencing the wind variation have been discussed: altitude, wind speed, geography, season and meteorological conditions.

3. Material and methods

3.1 The measurements

The basis for the present study is serial pilot-balloon ascents made with radar. The balloons were released at fixed time intervals. In most cases the time intervals were exactly one hour, in some cases they were exactly half an hour, but in a few cases they were between half an hour and two hours. In these cases wind vectors with an interval of one hour were obtained by interpolation. Consequently, the material only permits a study of the variability during intervals which are multiples of one hour.

The radar measures the slant range, elevation and azimuth of a reflector attached to the pilot balloon. The range and the angles are indicated by pointer instruments. The range is given in metres, the elevation in degrees and sixteenths of degrees, and the azimuth in artillery units, of which 6300 are equal to 360 degrees. The radar tracks the reflector automatically. The reflectors used were of corner type and had a reflection area equal to a medium-sized aircraft. Because the reflector rotates during its ascent, the reflection area varies, but the reflection minima are narrow and not disturbing in continuous tracking.

The pilot-balloon ascents were made at different places in the southern part of Sweden and mainly during the summer half-year (Table 1). The location of the places mentioned in the table are also shown in Fig. 3 together with some other places of interest in this study.

The number of individual pilot-balloon ascents within the different serial ascents varied between 2 and 19. The total number of individual pilot-balloon ascents was 776. The majority of the series or 49 per cent were made in situations with straight contour lines, 39 per cent were made in troughs and 12 per cent in ridges (see section 3.3 below). Some measurements were made for the specific purpose of studying wind variations, but the majority, however, were made for other purposes. However, all were made with the same type of radar equipment. Of course, the radar equipment and the persons who made the measurements were not the same on every occasion. The personnel had, however, a unitary training and the radar equipment used was serviced in the same way and with centralized administration. Therefore the collected material is estimated to be homogeneous.

TABLE 1. *Data for the ascents.*

Place	Lat. N	Long. E	Date		No. of serial ascents
Ytterskär	59°58′	18°52′	June, July, October	1953	7
Ytterskär			May–August	1954	12
Ytterskär			April–September	1955	11
Ytterskär			April–September	1956	13
Ytterskär			October	1957	11
Karlsborg	58°31′	14°30′	June–September	1953	9
Karlsborg			April	1956	4
Uppsala	59°53′	17°36′	July, September	1957	27
Gothenburg	57°44′	11°57′	June	1954	4
Bromma	50°20′	17°57′	Synoptic measurements for 1951–1953		

As the measurements were made only in the southern part of Sweden and during the summer, it was not possible to carry out a desirable investigation of the variability during other seasons and in other parts of the country.

The radar wind measurements were supplemented by synoptic radio wind measurements from the aerological station at Bromma in order to obtain wind values from heights above those obtained by radar and also in order to obtain measurements made at higher wind speeds than when the radar wind measurements were made.

3.2 Some theoretical considerations

A pilot balloon is affected by all motions in the atmosphere. By using pilot-balloon ascents for the determination of the variability, the question arises how well a balloon follows these motions. The inertia of the balloon, especially if it is fitted with a reflector, prevents the balloon from following the small-scale turbulent motions. It is difficult to arrive at a good understanding of how well the balloon follows other motions, but with some simplifying assumptions it is possible in a theoretical way to get an idea of the coupling between real air motions and the motions of the balloon.

In order to obtain such an understanding we shall follow some indications of Merbt (1957) and consider the conditions which exist when a pilot balloon, during its ascent, enters into a sharp delimited layer with great wind speed (sharp edge gust). Immediately after the balloon (with the horizontal speed u) has entered into the sharp edge gust (Fig. 4), in which the wind speed u_g is assumed to be horizontal and constant, its total speed relative to the air is v. The direction of v has the angle β with the direction of u_g. The vertical component of v is w (the rate of ascent of the balloon) and the horizontal component is $u_g - u$, i.e. the horizontal speed of the balloon relative to the air in the gust.

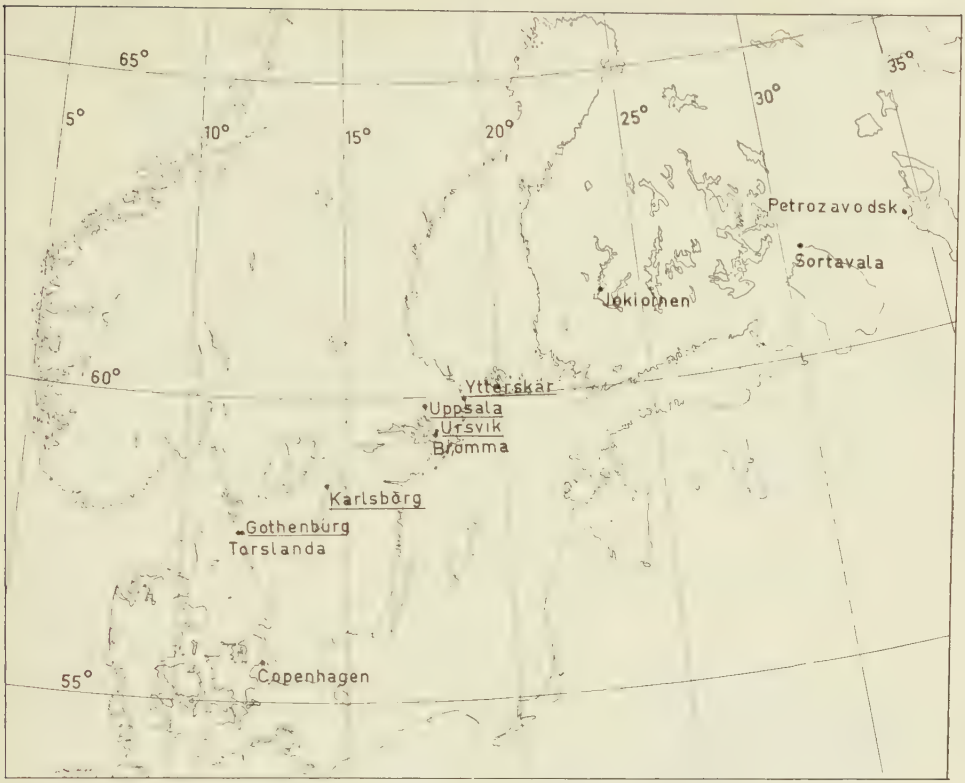

FIG. 3. Places actual in the present study. The names underlined indicate places where serial upper-wind measurements with radar were made.

According to aerodynamics (see, for instance, Hann–Süring, 1942, p. 485), the drag D of the balloon in the direction of v is defined by

$$D = c\pi r^2 \frac{\varrho}{2} v^2, \tag{1}$$

where c is the coefficient of drag of the balloon; r is the radius of the balloon; and ϱ is the density of air.

The drag in the direction of $u_g - u$ is, as Fig. 4 shows,

$$D\cos\beta \quad \text{or} \quad D\frac{u_g - u}{v}.$$

Consequently

$$m\frac{d(u_g - u)}{dt} + D\frac{u_g - u}{v} = 0,$$

where m is the mass of the balloon.

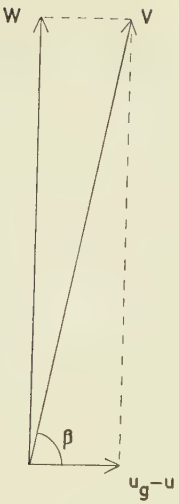

FIG. 4. Velocities affecting a pilot balloon in a sharp edge gust.

But u_g is assumed to be a constant, and therefore

$$-\frac{m}{D}\frac{du}{dt}+\frac{u_g-u}{v}=0. \tag{2}$$

According to equation (1),

$$\frac{m}{D}=\frac{2m}{c\pi r^2 \varrho v}\cdot\frac{1}{v},$$

or, if $2m/c\pi r^2 \varrho v$ is put equal to $\varkappa$,

$$\frac{m}{D}=\frac{\varkappa}{v}.$$

Thus equation (2) can be written

$$\varkappa\frac{du}{dt}+u-u_g=0.$$

Assuming $\varkappa$ to be a constant, integration gives

$$\frac{u}{u_g}=1-C\cdot\exp(-t/\varkappa), \tag{3}$$

where C is an integration constant.

The assumption that $\varkappa$ is a constant is only approximately correct. Concerning pilot balloons, however, w is much greater than u_g-u. Then, according to Fig. 4, $v=w$ and

$$\varkappa=\frac{2m}{c\pi r^2 \varrho w}. \tag{4}$$

TABLE 2. *ICAN values of density and air temperature for certain altitudes together with the values of viscosity corresponding to the temperatures.*

Altitude, km	ϱ, g cm^{-3}	t, °C	η, g cm^{-1} sec^{-1}
0	1223×10^{-6}	+15.0	1.75×10^{-4}
1	1113	+ 8.5	1.76
10	413	−50.0	1.44

Let us first consider $r^2 \varrho w$. According to Hann–Süring (1942, p. 485), $r^3 \sim \varrho^{-1}$ and $w \sim \varrho^{-1/6}$. Consequently, $r^2 \varrho w \sim \varrho^{1/6}$. Therefore $r^2 \varrho w$ varies only a little with height.

Now it remains to consider c, the coefficient of drag. It is a function of the Reynolds number R, as Fig. 5 shows. We must therefore make an estimation of the values of R which can occur during the ascent of the balloon. The following expression (see, for instance, Scorer, 1958, pp. 87–89) is valid for R:

$$R = \frac{2 r w \varrho}{\eta},$$

where η is the viscosity of the air.

In order to express ϱ and η as a function of height the ICAN atmosphere has been used. For certain altitude values Table 2 gives the corresponding ICAN values of ϱ and t. As η is a function only of the air temperature, the η-value corresponding to the temperature values of Table 2 have been obtained from a table in Landolt–Börnstein (1955, p. 604), giving the viscosity of the air as a function of the air temperature.

In the expression for R the radius r of the balloon also varies with height. As mentioned above, $r^3 \sim \varrho^{-1}$. From the values of Table 2 it follows that if r at a height of 1 km is r_1 and at 10 km is r_{10}, $r_{10} \approx 1.4 r_1$.

In our pilot balloon ascents r_1 was about 50 cm and w about 3 m/sec. Consequently $R_1 \approx 2 \times 10^5$ and $R_{10} \approx 10^5$.

With these values of R_1 and R_{10} we obtain from Fig. 5 the value of 0.5 for the coefficient of drag when $10^5 \leq R \leq 2 \times 10^5$.

From the discussion above it therefore seems legitimate to regard $\varkappa$ as a constant.

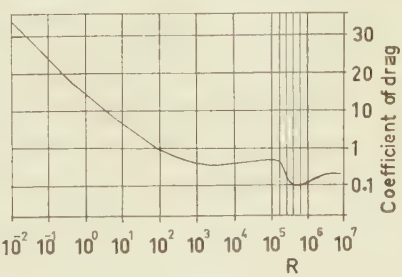

FIG. 5. Drag coefficient of a sphere as a function of the Reynolds number. (From Landolt–Börnstein, 1955.)

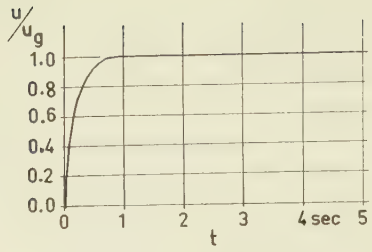

FIG. 6. The function $u/u_g = 1 - \exp(-t/0.2)$.

In order to apply the expression (3) to the pilot-balloon ascents, we need a numerical value of $\varkappa$. This can be obtained from the expression

$$\varkappa = \frac{2m}{c\pi r^2 \varrho w},$$

and the following data from our pilot-balloon ascents.

Weight of balloon and reflector	0.35 kg
Effective area of balloon and reflector	0.8 m^2
Rate of ascent	3 m/sec
Air density	1.2 kg/m^3
Coefficient of drag	0.5

We obtain a value of $\varkappa$ equal to 0.2.

With this value of $\varkappa$ the expression (3) is to be written

$$\frac{u}{u_g} = 1 - C \cdot \exp(-t/0.2). \tag{5}$$

From this expression we can calculate a value of t corresponding to $u/u_g = 1$, i.e. the time interval between that point of time when the balloon enters the sharp edge gust and until it has obtained the velocity u_g of the gust. The calculation is dependent on the value of C. From the assumptions made concerning u and u_g we obtain $0 \leqslant C \leqslant 1$. With $C = 1$ the expression (5) gives the largest value of t corresponding to $u/u_g = 1$.

The function $u/u_g = 1 - \exp(-t/0.2)$ is shown in Fig. 6. It is seen that $u/u_g = 1$ already after practically only one second, i.e. the pilot balloon follows the air motions with a drag of only one second.

Even if, because of the approximations made above and the assumption of a sharp edge gust, this time were to be ten times greater, we could still consider the balloon as following the air motions perfectly, as far as pilot-balloon ascents are concerned.

The conventional method of calculating wind values from pilot-balloon ascents also calls for some discussion. In order to obtain the velocity of the wind from a pilot-

balloon ascent, the position of the balloon must be determined at two points. To compute the velocity we calculate the projection of the balloon drift on the horizontal plane and divide it by the time. Thus the velocity obtained is a mean value taken within a certain volume of the air or, less exactly, within the layer which the balloon has passed. The following considerations illustrate the importance of these mean values.

The height of the lower boundary of the layer of interest is z_0 and of the upper boundary z_1. The horizontal component $\mathbf{u}$ of the balloon velocity is a function $\mathbf{u}(z)$ of the height. Thus the mean wind velocity in the layer is

$$\overline{\mathbf{V}} = \frac{1}{z_1 - z_0} \int_{z_0}^{z_1} \mathbf{u}(z)\, dz.$$

As the velocity is obtained by the projection $\mathbf{p}$ of the balloon path, which also is a function $\mathbf{p}(z)$ of the height, the function $\mathbf{u}(z)$ must be converted into $\mathbf{p}(z)$.

$$\mathbf{u}(z) = \frac{d\,\mathbf{p}(z)}{dt} = \frac{d\,\mathbf{p}(z)}{dz} \cdot \frac{dz}{dt}.$$

Thus

$$\overline{\mathbf{V}} = \frac{1}{z_1 - z_0} \int_{z_0}^{z_1} \frac{d\,\mathbf{p}(z)}{dz} \frac{dz}{dt}\, dz.$$

The integral contains two variables $\mathbf{p}$ and t. The latter must be removed, which can be done by considering dz/dt (the rate of ascent) as constant and equal to $(z_1 - z_0)/(t_1 - t_0)$.

Thus

$$\overline{\mathbf{V}} = \frac{1}{z_1 - z_0} \cdot \frac{z_1 - z_0}{t_1 - t_0} \int_{z_0}^{z_1} \frac{d\,\mathbf{p}(z)}{dz}\, dz = \frac{\mathbf{p}_1 - \mathbf{p}_0}{t_1 - t_0}.$$

This is the expression conventionally used to obtain the mean velocity in a layer.

Thus two things concerning pilot-balloon ascents are to be noted. *First*, the balloon ascent rate is assumed to be constant and this is valid not only for single-theodolite measurements but also for every type of pilot-balloon measurements. There are many cases when the rate of ascent of the balloon is not constant (balloon leakage, vertical currents, etc.). If large layers are chosen the probability is greater that the rate of ascent is not constant. The *second* point is concerned with the practice, common in meteorology, of ascribing the mean velocity to the midpoint of the layer. The integration merely gives the mean value without stating where in the layer this value occurs. However, there is nothing to be done about this.

Consequently, when pilot-balloon ascents are used for the investigation of the variability, it is essential to remember that obtained wind values do not refer to a certain level but to a certain layer. It is also essential that the investigation shall be carried out only by comparing data from layers of the same thickness.

3.3 Treatment of obtained wind values

From the data obtained by the radar equipment the wind values were computed by the use of an electronic computer. For each layer which the balloon had passed between two consecutive readings, the wind velocity components were computed and also the altitude of the upper and lower boundary of the layer. The components were positive when they referred to northerly and easterly directions and negative when they referred to southerly and westerly directions.

As the wind measurements used in this study were made on many different occasions and at many places, the rate of ascent of the balloons of different serial ascents was not always the same. The pilot balloons which were released within one and the same series of ascents had, however, approximately the same rate of ascent. The layers defined by two consecutive balloon positions could therefore be of different thickness in measurements from different serial ascents. As this investigation is based on comparisons between different wind values, it was essential, as mentioned above, that the wind values under consideration should refer to layers of the same thickness. For every individual pilot-balloon ascent, therefore, two curves were drawn, one giving the northerly-southerly wind component as a function of altitude, the other the easterly-westerly component as a function of altitude. From these curves two mean values, one of the northerly-southerly wind components, the other of the easterly-westerly wind components within layers with a thickness of 500 m were integrated graphically. The method involves a smoothing, but is necessary in order to obtain wind values from layers with the same thickness.

In order to limit the calculation work, not all the 500 m layers which the measurements embraced were treated, but only those between the altitudes 1.0–1.5, 3.0–3.5, 5.0–5.5, 7.0–7.5 and 9.0–9.5 km and in two special cases the layers between the altitudes 1.5–2.0 and 4.5–5.0 km. The mean wind components obtained within these layers define the wind vectors which are the *basic data* used in order to calculate the variability values.

The basic wind vectors, which could be obtained from one series of pilot-balloon ascents, were not numerous enough to permit a statistical treatment of the vectors in order to obtain a value of the variability. From different serial ascents the basic wind vectors from 500 m layers situated at the same level were therefore put together to form large series. It is essential to put together wind vectors only from layers situated at the same level, since wind vectors from a layer situated at a high level are often affected by larger errors than wind vectors from a layer situated at a low level. When combining the basic wind vectors from several serial ascents, consideration was taken of the factors discussed in section 2 above, viz. influences exerted by synoptic situation and wind speed. The serial pilot-balloon ascents were therefore first classified with regard to the synoptic situation in which they were made. A

classification of synoptic situations is in itself a problem. Therefore a classification in rather rough outlines was made with the following groups: ridges, troughs, and situations with straight contour lines. Even with this rough classification there were cases in which it was uncertain how to classify a series of pilot-balloon ascents. When, for instance, a ridge was situated with its peripheral parts above the place of measurement, it was sometimes uncertain when the wind speed was high, whether the measurements should be classified as being made in a ridge or in a situation with straight contour lines.

Then, all series made in ridges were put together, and similarly, all series made in troughs and all series made in situations with straight contour lines. In doing this it was in some cases necessary to classify the low layers in one series as being made, for instance, in a trough and the high layers as being made in a situation with straight contour lines. Within every one of the three groups of synoptic situations the series of ascents were grouped into certain wind speed intervals (< 5, 5–10, 10–15, etc. m/sec). When the speed varied with the altitude, different layers of the same series were assigned to different speed intervals. The formation into groups can easily be surveyed in Tables 5*a–c* (pages 39–42). These tables contain a summary of obtained values of wind variability distributed upon different synoptic situations, wind-speed intervals and layers.

It is questionable whether it is permissible from a statistical point of view to put together serial pilot-balloon ascents from different occasions and different places. Concerning the fact that the measurements were made at different occasions, this was assumed to be of no account, since practically all of the measurements were made during the summer half-year and were combined with regard to the synoptic situation. To combine measurements from different places can only be correct if there is no fundamental difference between the variability at different places. In order to get a better idea of this question an investigation by Brooks *et al.* (1950) was used. In this work the standard vector deviation of individual winds about their vector mean is mapped. From the maps which apply to the months March–November that part which comprises the north-western part of Europe is reproduced in Fig. 7. The numerals in Fig. 7 give the standard vector deviation in knots.

From Fig. 7 it follows that the standard vector deviation varies very little from place to place within the part of Sweden in which the wind measurements used in this investigation were made. As the standard vector deviation given on the maps of Fig. 7 is the same as the standard vector deviation σ given by Durst (1954) and discussed in section 1 above, it follows from the expression

$$\sigma_t = \sigma \sqrt{2\,(1 - r_t)}$$

that σ_t—the variation of wind with time—also varies very little for a given altitude and a given value of t within the actual area.

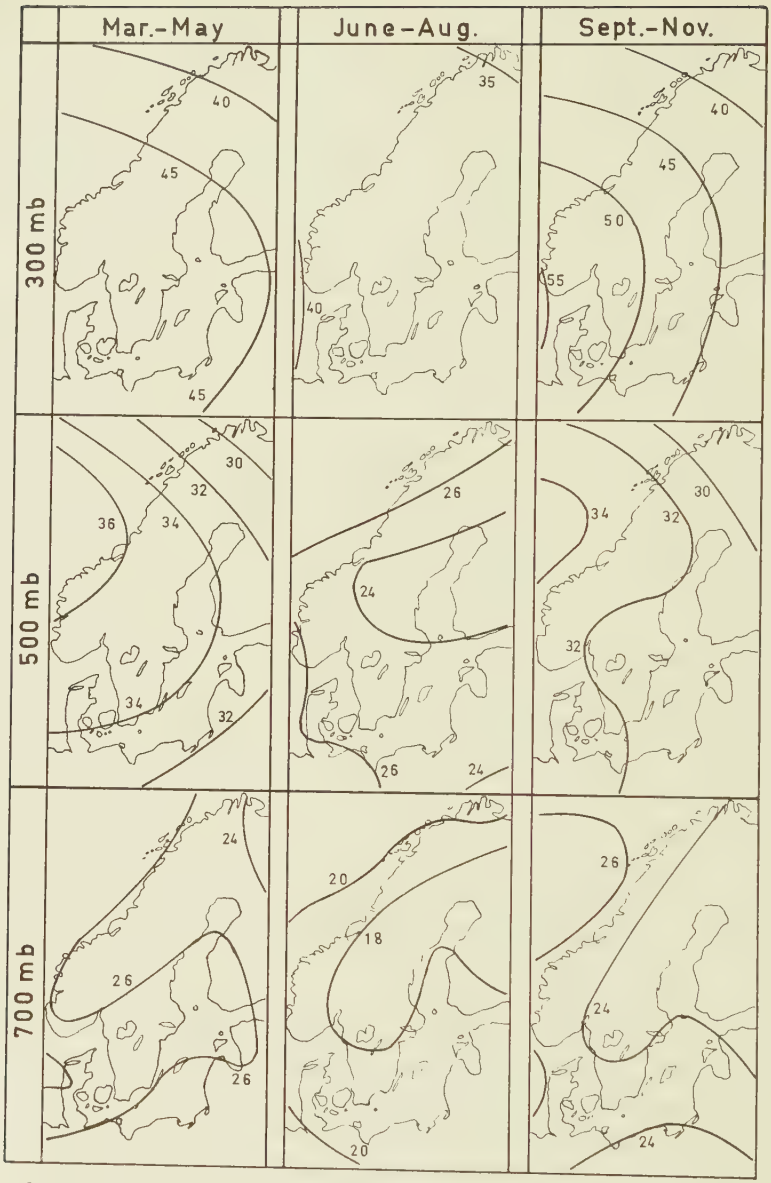

FIG. 7. Standard vector deviation (knots) of individual winds about their vector mean. (From Brooks *et al.*, 1950.)

It is true that σ_t is not the same as our wind variability, as σ_t also contains those variations in the wind which are caused by changes in the synoptic situation. However, this does not affect the discussion here and it therefore seems legitimate to combine pilot-balloon ascents from different places within the area in question.

3.4 The standard vector deviation as an expression of the wind variability

If the basic wind vectors are indicated by **W**, we can write a series composed of vectors from different serial pilot-balloon ascents according to the principles given in section 3.3 above as follows.

$$\mathbf{W}_{11},\ \mathbf{W}_{12},\ \ldots,\ \mathbf{W}_{1i},\ \ldots,\ \mathbf{W}_{1n},$$
$$\mathbf{W}_{21},\ \mathbf{W}_{22},\ \ldots,\ \mathbf{W}_{2i},\ \ldots,\ \mathbf{W}_{2m},$$
etc.

Here the first index indicates the serial ascents from which the vectors were obtained and the second the individual vectors. The time interval between the vectors of each series was—according to section 3.1 above—always one hour. By taking pairs of vectors **W** with a given time interval (1, 2, etc. hours), and subtracting vectorially the *later* vector from the *earlier*, a series of *difference vectors* was obtained. These difference vectors characterize the wind variability connected with the chosen time interval. Of course, it is necessary in practice to use some statistical parameter in order to characterize the series of difference vectors. The vector mean is not a useful parameter, since in a long series of difference vectors of the type used here, the vector mean tends to be zero or near zero although every individual difference vector may have a considerable module. The standard vector deviation of the difference vectors about their vector mean is, however, not zero. Therefore—and also for other reasons—*the standard vector deviation was used as an expression of the wind variability.*

An expression of the standard vector deviation σ of a series of N difference vectors $\mathbf{V}_1, \ldots, \mathbf{V}_i, \ldots, \mathbf{V}_N$ can be obtained in a similar way as the standard deviation of a linear distribution (see, for instance, Brooks & Carruthers, 1953, p. 196). Assume that the module of the vector mean of the difference vectors $\mathbf{V}_1, \ldots, \mathbf{V}_i, \ldots, \mathbf{V}_N$ is V_R. Then the deviation v_i of a difference vector $\mathbf{V}_i$ from the vector mean is defined by

$$v_i^2 = V_R^2 + V_i^2 - 2V_R V_i \cos\varphi_i, \tag{6}$$

where φ_i is the angle between $\mathbf{V}_r$ and $\mathbf{V}_i$. Analogous to the definition of the standard deviation of a scalar quantity we have

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} v_i^2.$$

Summing over N vectors we obtain from equation (6)

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} v_i^2 = \frac{1}{N}\sum_{i=1}^{N}(V_R^2 + V_i^2 - 2V_R V_i \cos\varphi_i)$$

$$= V_R^2 + \frac{1}{N}\sum_{i=1}^{N} V_i^2 - \frac{1}{N}2V_R\sum_{i=1}^{N} V_i \cos\varphi_i.$$

Now $V_i \cos \varphi_i$ is the projection of the vector $\mathbf{V}_i$ on the direction of V_R. The sum of all these projections is equal to the vector sum of all vectors $\mathbf{V}_i$. Consequently $\sum_{i=1}^{N} V_i \cos \varphi_i$ is equal to NV_R and

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} V_i^2 - V_R^2.$$

This expression is valid for all kinds of vector distributions.

The standard deviation of a scalar quantity is a standard length taken as a measure of the dispersion about the scalar mean. Analogous to this, the standard deviations σ_x and σ_y of the components V_{ix} and V_{iy} of the vectors $\mathbf{V}_i$ in two directions at right angles define an ellipse (V_{ix} and V_{iy} are assumed to be independent). In order to put the standard vector deviation into practice we must know the proportion between those vectors which end within the ellipse and all vectors. However, this proportion can only be calculated if the analytic expression of the frequency surface of the vectors is known.

Now a vector $\mathbf{V}_i$ is the difference between two wind vectors. Individual wind vectors at a point in the free air tend, however, according to Brooks *et al.* (1946), to be normally distributed about the vector mean wind. Any linear function of independent normally distributed variables is, however, also normally distributed (Cramer, 1949, p. 99). Therefore also the difference vectors $\mathbf{V}_i$ tend to be normally distributed about their vector mean.

It is a restriction connected with the statement that wind vectors at a point in the free air are normally distributed. It would not hold if the period included in a frequency distribution covered the transition from one prevailing wind to another. It would therefore not in general apply to frequency distributions for a whole year, but a distribution for a part of a year (spring, summer, etc.) is consistent with a normal distribution as confirmed by statistical tests (Brooks *et al.*, 1946).

The conclusion that the vectors $\mathbf{V}_i$ are normally distributed implies that the wind vectors from which $\mathbf{V}_i$ are obtained are independent variables. This is only correct to a certain extent. The assumption of a normal distribution is, however, the most simple one to make and it would therefore be very favourable if the normal distribution could be used. In order to examine to what extent this assumption is relevant the following experimental test was made.

The test is worked out for the difference vectors $\mathbf{V}_i$ used for the calculation of the variability in troughs and for the height interval 3.0–3.5 km, with a wind speed < 5 m/sec and the time interval of 1 hour (Table 5*b*). The components of the vectors $\mathbf{V}_i$ (positive when they refer to northerly and easterly directions and negative when they refer to southerly and westerly directions) are V_{ix} and V_{iy}. They are divided into speed classes, as Table 3 shows. A criterion of the normality of the V_{ix} and V_{iy} series can be obtained by plotting the values "Frequency sum: percentage of N" from Table 3 on a "probability paper" (Brooks & Carruthers, 1953, p. 101). If the points

TABLE 3. *Frequency distribution of the components of the vectors* $\mathbf{V}_i$.

Speed (m/sec)	Central value	The V_{ix} series			The V_{iy} series		
		Frequency	Frequency sum	Frequency sum: % of N ($N = 149$)	Frequency	Frequency sum	Frequency sum: % of N ($N = 149$)
−3.5–−2.5	−3	—	—	—	1.0	1.0	0.67
−2.5–−1.5	−2	11.5	11.5	7.7	10.0	11.0	7.4
−1.5–−0.5	−1	26.0	37.5	25.2	34.5	45.5	30.5
−0.5–0.5	0	67.5	105.0	70.5	71.5	117.0	78.5
0.5–1.5	1	36.5	141.5	95.0	26.0	143.0	96.0
1.5–2.5	2	6.5	148.0	99.33	6.0	149.0	100.0
2.5–3.5	3	1.0	149.0	100.0	—	—	—
		Mean: 0.02 m/sec			Mean: −0.13 m/sec		
		S.D.: 0.98 m/sec			S.D.: 0.95 m/sec		

obtained by the plotting are on a straight line, there is a normal distribution. Fig. 8 shows that the points obtained from Table 3 are approximately situated on a straight line. Therefore both the V_{ix} and the V_{iy} series are practically normally distributed and consequently the distribution of $\mathbf{V}_i$.

From Table 3, "Frequency" and "Central value", the mean of the V_{ix} series was calculated at 0.02 m/sec and the mean of the V_{iy} series at −0.13 m/sec. From these values the module V_R of the mean vector of the vectors $\mathbf{V}_i$ was calculated at $\sqrt{0.02^2 + 0.13^2} = 0.13$ m/sec.

From the facts given above it therefore seems legitimate to consider the distribution of the difference vectors $\mathbf{V}_i$ to be normal distributed with a zero mean.

From "Frequency" and "Central value" the standard deviation of the V_{ix} series was calculated at 0.98 m/sec and of the V_{iy} series at 0.95 m/sec. Thus the standard deviations of the components of the vectors $\mathbf{V}_i$ are approximately identical. It therefore seems legitimate to consider the distribution of the vectors $\mathbf{V}_i$ as circular. This means that the area defined by the standard vector deviation is a circle with a radius equal to the standard vector deviation. In this case the proportion between those vectors which end within the circle and all vectors is 63 per cent.

The test above is not, of course, a valid proof that such series of difference vectors are always normal and circular distributed with a zero mean. However, it supports the view that this assumption is true.

In order to illustrate the practical application of the standard vector deviation as a measure of the wind variability, the following discussion is undertaken. Let us first remind that the letter s is used for indicating the standard vector deviation. This is done because the letter σ used above normally denotes the standard vector deviation of a population, whereas this investigation only deals with samples. Assuming that

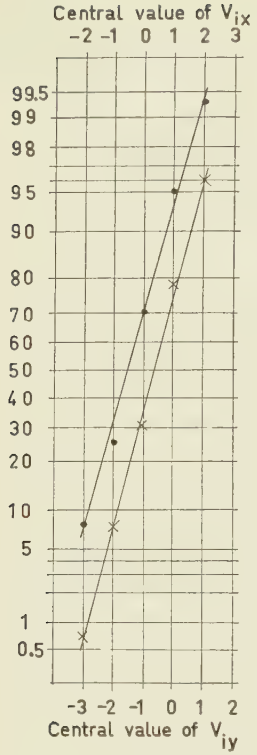

FIG. 8. The values from Table 3 plotted in a "probability diagram". ● represents the V_{ix} series; × represents the V_{iy} series.

we have a normal and circular distribution, Fig. 9 is a graphical representation of s. In Fig. 9a **V** is a wind vector obtained at a given height and at a given time. $\mathbf{V}_R$ is the mean of all vector differences calculated for a given time interval t and s is the standard vector deviation of the distribution of these vector differences. As we have assumed a distribution with a zero mean, the vector $\mathbf{V}_R$ is zero. The circle with the radius s will therefore be about the original vector **V** as shown in Fig. 9b.

This circumstance is of great importance in the practical application of the wind variability. Suppose that a wind vector **V** is obtained at a given time and height and that the variability s for a given time interval t is known. From Fig. 9b it then follows that we can predict that the wind vector which occurs after the time interval t in

TABLE 4. *Proportions for multiples of the standard vector deviation.*

Per cent	25	50	63	75	80	85	90	95	99
Multiples of $\sigma(s)$	0.54	0.83	1.0	1.18	1.27	1.38	1.52	1.73	2.15

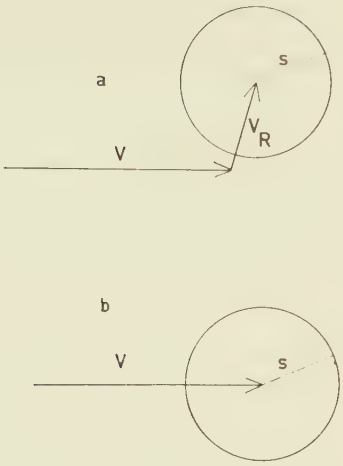

Fig. 9. Representation of the standard vector deviation.

63 per cent of all cases will fall with its end within the circle of radius s. If the value of 63 per cent is not suitable in a special case, other proportions for multiples of s are given in Table 4.

3.5 Errors

(a) *Errors of the wind measurements*

In wind measurements where an anemometer can be used, the error of a measurement can be determined by comparisons between the anemometer and a precision instrument. This can be done, first in a wind tunnel, secondly during the measurements. In pilot-balloon ascents this is impossible except for low heights where comparisons can be made with anemometers. Other methods must therefore be used. For instance, one can have two radar sets tracking the same balloon. It is then possible to calculate the measurement errors at the same time as the variability measurements are made and under identical conditions. It was not, however, possible to use this method for this investigation. The measurement errors have therefore been calculated in the theoretical way which will be described below.

Cartesion co-ordinates are used. The axis of x is horizontal, and directed to the north, the axis of y is horizontal and directed to the east, and the axis of z is vertical and directed upward. The slant range from the radar set to the pilot balloon is r, the elevation is e and the azimuth is a. Then the position of the balloon is defined by

$$\left.\begin{aligned} x &= r \cos e \cos a, \\ y &= r \cos e \sin a, \\ z &= r \sin e. \end{aligned}\right\} \qquad (7)$$

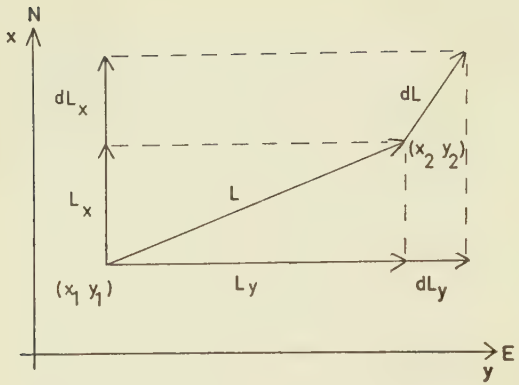

FIG. 10. Projection on a horizontal plane of a pilot-balloon drift between two consecutive readings.

If the standard errors of the co-ordinates are Δx, Δy and Δz, then

$$\Delta x = \pm\left[\left(\frac{\partial x}{\partial r}dr\right)^2 + \left(\frac{\partial x}{\partial e}de\right)^2 + \left(\frac{\partial x}{\partial a}da\right)^2\right]^{\frac{1}{2}} \tag{8}$$

and similar expressions for Δy and Δz where dr, de and da are the standard errors of r, e and a.

After the partial derivates have been calculated from (7) and inserted in the radical quantities, we get

$$\left.\begin{aligned}
\Delta x &= \pm[\cos^2 e \cos^2 a\,(dr)^2 + r^2 \sin^2 e \cos^2 a\,(de)^2 + r^2 \cos^2 e \sin^2 a\,(da)^2]^{\frac{1}{2}},\\
\Delta y &= \pm[\cos^2 e \sin^2 a\,(dr)^2 + r^2 \sin^2 e \sin^2 a\,(de)^2 + r^2 \cos^2 e \cos^2 a\,(da)^2]^{\frac{1}{2}},\\
\Delta z &= \pm[\sin^2 e\,(dr)^2 + r^2 \cos^2 e\,(de^2]^{\frac{1}{2}}.
\end{aligned}\right\} \tag{9}$$

Now let L (Fig. 10) with the error dL be the distance between two consecutive projections $(x_1 y_1)$ and $(x_2 y_2)$ of the pilot balloon in the xy plane. L_x and L_y are the components of L, and dL_x and dL_y are the errors of L_x and L_y.

Then from Fig. 10 we get

$$(dL)^2 = (dL_x)^2 + (dL_y)^2.$$

But $L_x = x_2 - x_1$ and $L_y = y_2 - y_1$.

Consequently, dL_x and dL_y are obtained by

$$dL_x = \pm[(\Delta x_1)^2 + (\Delta x_2)^2]^{\frac{1}{2}} \quad \text{and} \quad dL_y = \pm[(\Delta y_1)^2 + (\Delta y_2)^2]^{\frac{1}{2}}.$$

The slant range, the elevation and the azimuth usually do not vary much between two consecutive readings, and so, according to expression (9), $\Delta x_1 \approx \Delta x_2$ and $\Delta y_1 \approx \Delta y_2$. Both Δx_1 and Δx_2 may therefore be replaced by Δx; Δy_1 and Δy_2

by Δy. The approximation made above does not hold, of course, if the wind speed and/or the rate of ascent of the balloon is very large.

Consequently $\quad dL_x = \pm \Delta x \sqrt{2} \quad$ and $\quad dL_y = \pm \Delta y \sqrt{2}$

and

$$(dL)^2 = 2\,[(\Delta x)^2 + (\Delta y)^2]. \tag{10}$$

Now let V be the wind speed and t the time between two consecutive readings, during which the projection of the balloon moves from $(x_1\, y_1)$ to $(x_2\, y_2)$.

Then $\quad V = \dfrac{L}{t} \quad$ and $\quad dV = \dfrac{dL}{t},$

and with expression (10)

$$(dV)^2 = \frac{2}{t^2}\,[(\Delta x)^2 + (\Delta y)^2].$$

With the expressions for Δx and Δy from (9) we get

$$(dV)^2 = \frac{2}{t^2}\,[(dr)^2 \cos^2 e + (de)^2\, r^2 \sin^2 e + (da)^2\, r^2 \cos^2 e]. \tag{11}$$

Here dV is the standard error of an individual wind vector.

Before the expression (11) could be used to calculate dV the numerical values of dr, de and da must be known. Received values of these quantities were, however, valid only for the minimum performances of the radar sets.

In order to have a check on the values of dr, de and da and on the training and ability of the personnel, the same pilot balloon was tracked by four radar sets at the same time. These measurements were made at Ytterskär, one on June 25 and two on October 31, 1957. The radar sets were located very near one another and the measurements were made in the same way as by the variability measurements. Simultaneous readings were obtained by sending simultaneous signals to the operators of the four sets. On June 25 the measurements were made up to an altitude of about 4 km. The wind speed was 5 m/sec in the lower levels and increased with height to 8 m/sec. On October 31 the measurements were made up to an altitude of about 5 km. The wind speed was 10 m/sec in the lower levels and increased with height to 20 m/sec. For each layer, which the balloon passed between two consecutive readings, the N–S component and the E–W component of the wind vector was calculated trigonometrically, separately for each of the four radar sets. The standard error of the wind vector of each layer was calculated. It was found to be about half of the error obtained from expression (11) for corresponding values of slant range and elevation by using the received values of dr, de and da. This result was estimated to be representative of all radar wind measurements used in this study, as the radar sets used were of the same type as the "control sets" (about 50 per cent of the variability measurements were

in fact made with the "control sets"). Moreover, all the measurements were made by persons of similar training and ability. In what follows, the standard error of an individual wind vector obtained by radar is therefore assumed to be $dV/2$.

Wind values from the aerological station at Bromma were also used. As these measurements were made with a radio-direction-finding equipment in connection with a radiosonde (radio wind), the errors connected with this method are also briefly discussed. As the height z is measured by the radiosonde, the Cartesian co-ordinates of the projection of the balloon are

$$x = z \cot e \cos a,$$

$$y = z \cot e \sin a.$$

In a similar way as in the discussion above, we obtain for the standard error dV_1 of an individidual wind vector the expression

$$(dV_1)^2 = \frac{2}{t^2}[\cot^2 e\,(dz)^2 + z^2 \sin^{-4} e\,(de)^2 + z^2 \cot^2 e\,(da)^2],$$

or, according to W.M.O. (1954),

$$(dV_1)^2 = \frac{2}{t^2}[Q^2\,(dz)^2 + z^2\,(1+Q)^2\,(de)^2 + z^2 Q^2\,(da)^2], \tag{12}$$

where $Q = \overline{V}/\bar{v} = \cot e$; $\overline{V}$ is the magnitude of the mean wind vector and $\bar{v}$ is the mean rate of ascent of the balloon both taken between the ground and height z.

With $de = da = 0.1°$, $t = 60$ seconds and on the assumption that dz is equal to the height error corresponding to a pressure error of the radiosonde of one millibar, the expression (12) was used to obtain the measurement errors.

When the expression (12) was used to estimate the values of dV_1 in a concrete case, Q was calculated on the assumption that $\overline{V}$ had the same value as the wind speed at height z in consideration and that $\bar{v}$ was equal to 6 m/sec. Of course this assumption regarding $\overline{V}$ gives slightly too high values of Q and therefore also too high values of dV_1. However, this is probably compensated for by the assumption of the rather low value of 0.1° for de and da.

A discussion of the altitude errors is also important, as they affect the possibility of working with layers of the same thickness and altitude above the ground by computing the wind variability. The layers which are considered in what follows are defined by the altitude differences 1.0–1.5, 1.5–2.0, 3.0–3.5, 4.5–5.0, 5.0–5.5, 7.0–7.5 and 9.0–9.5 km. According to the elevation angles and the slant ranges obtained by the measurements, the *maximum* altitude errors in the boundaries of the layers are calculated at 2 per cent of the altitude in the lowest layer and at 1 per cent in the other layers. It seems legitimate to overlook these errors in what follows.

(b) *Errors and corrections of calculated values*

According to the principles given in sections 3.3 and 3.4 above, we have series of basic wind vectors, where all series refers to the same type of synoptic situation, the same layer, and the same wind speed interval. Each of the basic vectors is a mean value of individual wind vectors obtained directly from the measurements. The standard error dW of such a basic vector is assumed to be equal to the arithmetic mean of the standard errors of the individual wind vectors from which it is obtained.

The vectors $$\mathbf{W}_{11}, \ldots, \mathbf{W}_{1i}, \ldots, \mathbf{W}_{1n},$$

with the standard errors $dW_{11}, \ldots, dW_{1n}$, may indicate one of the series of basic vectors. From this series we produced a series of difference vectors

$$\mathbf{V}_1, \ldots, \mathbf{V}_i, \ldots, \mathbf{V}_N,$$

where $$\mathbf{V}_i = \mathbf{W}_{1i} - \mathbf{W}_{1i+1}.$$

The standard error dV_i of a difference vector $\mathbf{V}_i$ is obtained by

$$dV_i = \pm [(dW_{1i})^2 + (dW_{1i+1})^2]^{\frac{1}{2}}.$$

The vectors $\mathbf{W}$ were, however, obtained from the same layer and belong to the same wind speed interval. They were also obtained from pilot-balloon ascents where all of the pilot balloons had approximately the same rate of ascent. Therefore $dW_{1i} \approx dW_{1i+1} \approx dW_i$ and consequently $dV_i \approx dW_i \sqrt{2}$.

Each of the difference vectors obtained may be regarded as the vector sum of the true difference vector due to the wind variability and the error $dW_i\sqrt{2}$. The square of the standard vector deviation of a sum of two uncorrelated terms is, however, equal to the sum of the squares of the standard vector deviation of each of the terms (see, for instance, Yule & Kendall, 1950, p. 326). Assuming the true variability and the errors to be uncorrelated, we have

$$\left(\frac{1}{N}\sum_{i=1}^{N} V_i^2 - V_R^2\right) = s^2 + \left[\frac{1}{N}\sum_{i=1}^{N} (dW_i\sqrt{2})^2 - (dW_R)^2\right],$$

where s is the standard vector deviation of the true difference vectors and dW_R is the module of the vector mean of the errors. Within the error vector distribution, however, the modules of the vectors are all of approximately the same magnitude and no special direction is predominant why dW_R tends to be zero. Thus, the expression given above enables us to eliminate the measurement errors. The expression mentioned may also be written

$$s_t^2 = s^2 + s_m^2,$$

where s_t is due to the true wind variability and errors and s_m to the errors of the measurements.

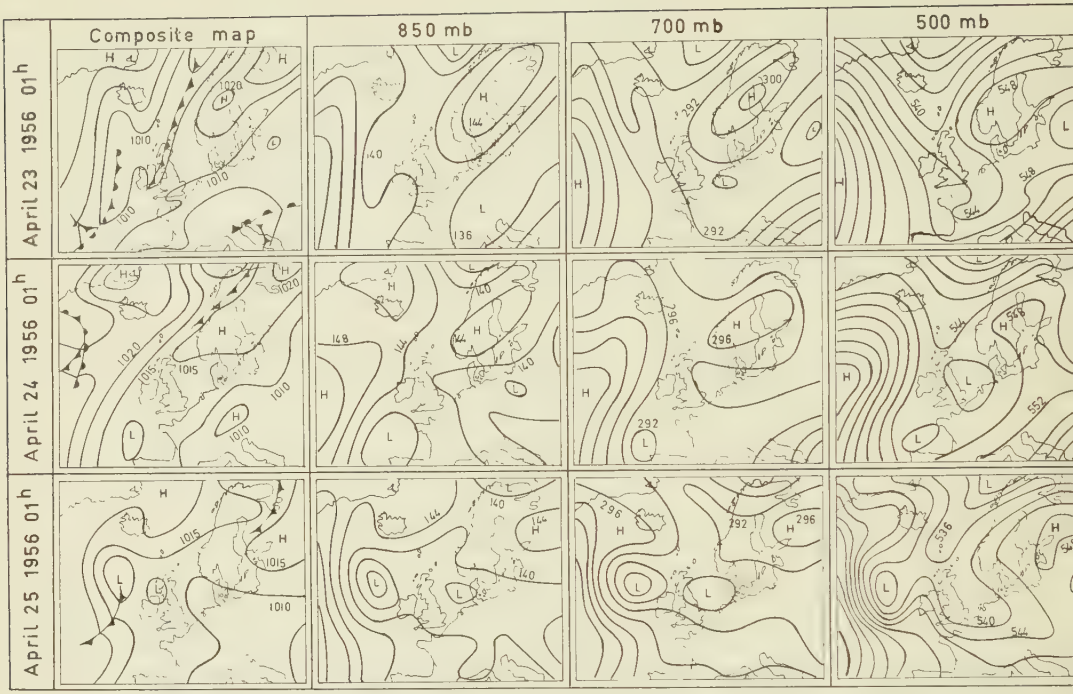

FIG. 11. Example of synoptic situation (ridge) when pilot-balloon ascents were made.

The number of ascents available for calculating the value of s diminishes with increasing time intervals. Especially by considering long time intervals the obtained value of s must be regarded as obtained from a small sample. s therefore involves an error, if we take it to be an estimate of the standard deviation of the population from which the sample was drawn. A closer estimate is obtained by using $s' = s\sqrt{N/N-1}$ instead of s (see, for instance, Cramér, 1949, p. 167), where N is the number of difference vectors from which s is calculated.

The standard error of s' is, according to Yule & Kendall (1950, p. 443), equal to $\pm s'/\sqrt{2N}$. Consequently, we may assume that our sought variability value lies in the range $s' \pm s'/\sqrt{2N}$.

4. Wind-variability values obtained

4.1 Meteorological conditions during some serial pilot-balloon ascents

The conception of wind variability relates to a wind field which on the upper-air maps appears to be quasi-stationary. Therefore it ought to be shown that the wind field also was quasi-stationary in this sense during the periods when the pilot-balloon

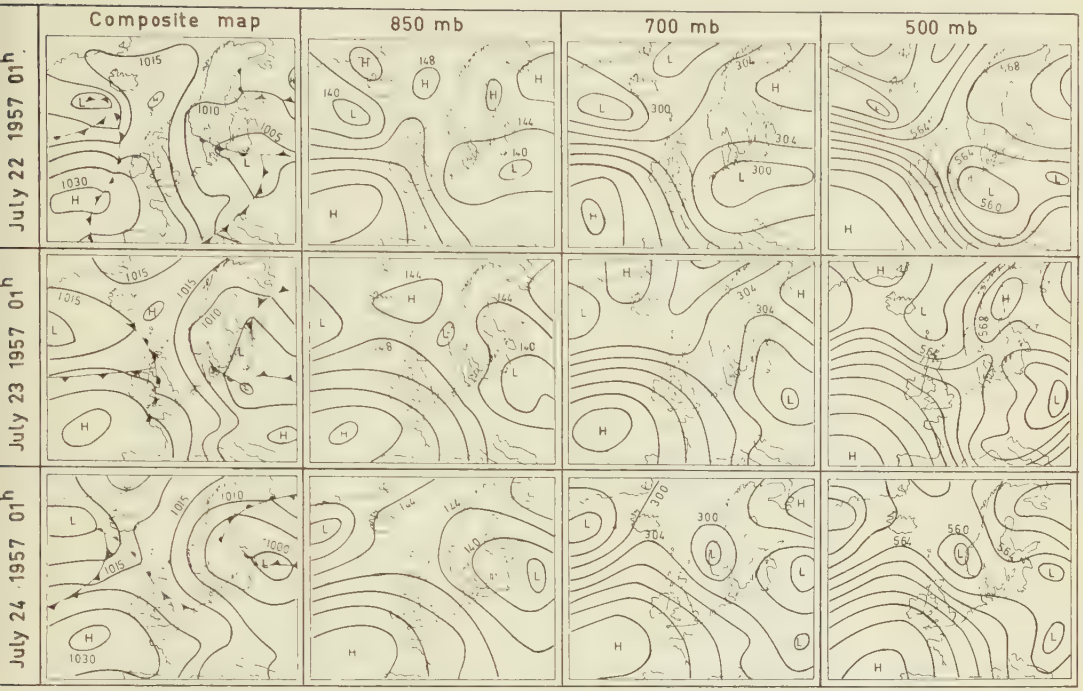

Fig. 12. Example of synoptic situation (trough) when pilot-balloon ascents were made.

ascents used in this study were made. In the sense of wind variability it may be permissible to define a quasi-stationary wind field as a field where the wind speed and direction are not markedly influenced by the movement and changes in intensity of cyclones and anticyclones and phenomena associated with these. Therefore, in this case a series of upper-air maps is sufficient in order to determine if a wind field is quasi-stationary or not.

We cannot here in detail specify all the different synoptic situations in which our pilot-balloon ascents were made. The account must be limited to some examples.

Fig. 11 shows the composite map[1] and the upper-air maps of 850 mb, 700 mb and 500 mb, all of 01^h for the period April 23–25, 1956. The synoptic situation illustrated by the maps was classified as a ridge. During this period serial pilot-balloon ascents were made at Karlsborg, between 22^h on April 23 and 05^h on April 24, and between 20^h on April 24 and 05^h on April 25. Only very slow changes occurred. No fronts passed the southern part of Sweden.

Fig. 12 shows the same types of maps as in Fig. 11 for the period July 22–24, 1957.

[1] The term composite map, introduced by T. Bergeron, is a more adequate name for weather map.

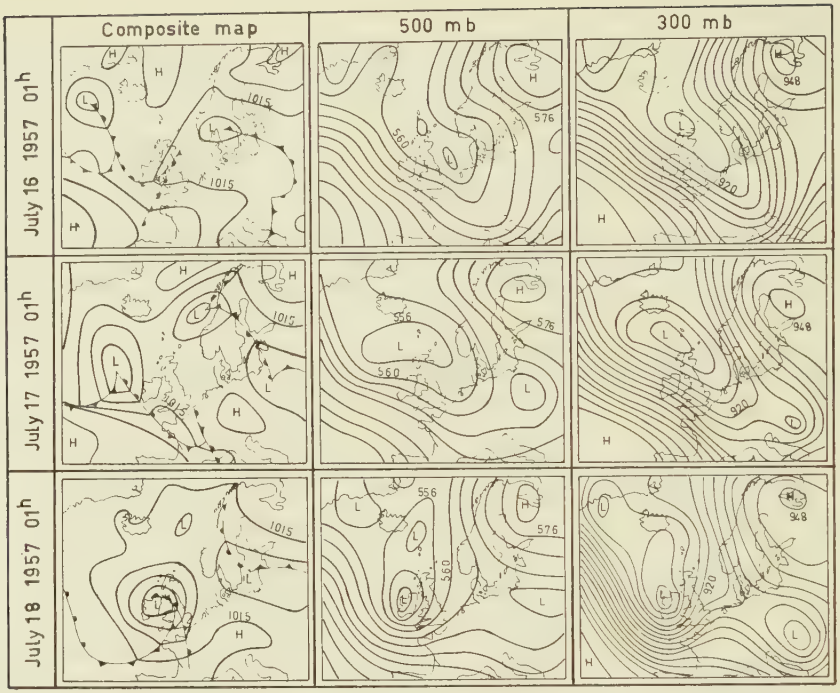

Fig. 13. Example of synoptic situation (straight contour lines) when pilot-balloon ascents were made.

The synoptic situation was classified as a trough. During this period serial pilot-balloon ascents were made at Uppsala daily between 08^h and 16^h. Only very slow changes occurred. No fronts passed during the measurement periods.

Fig. 13 shows the composite maps and the upper-air maps of 500 mb and 300 mb for the period July 16–18, 1957. The synoptic situation was classified as a situation with straight contour lines. During this period serial pilot-balloon ascents were made at Uppsala between 10^h and 16^h on July 16 and between 08^h and 16^h on July 17. No fronts passed during the measurement period. The jet stream over Scandinavia was unaltered and, as can be seen from the maps, it was not influenced by the movement of the cyclone west of the British Isles.

4.2 Discussion of obtained values

Note first the data concerning *ridges* (Table 5*a*). The highest level from which data were obtained was the 4.5–5.0 km layer when the wind speed was 5–10 m/sec and the 3.0–3.5 km layer when the wind speed was <5 m/sec. The reason why data were obtained only from these relatively low levels is that ridges do not reach very high levels.

TABLE 5*a*. *Wind variability in ridges.*

Wind speed (m/sec)	Layer (km)	Time interval (hours)	No. of synoptic situations	No. of difference vectors	Variability (m/sec) corrected for S.E.	Serial wind measurements from
< 5	1.0–1.5	1	6	45	0.2–0.6	Ytterskär
		2	6	35	0.6–1.0	
		3	6	29	0.6–1.3	
		4	5	21	0.5–1.1	
		5	5	14	1.0–1.7	
< 5	3.0–3.5	1	2	26	—[a]	
		2	2	21	—[a]	
		3	2	20	0.7–1.2	
		4	2	15	0.7–1.4	
		5	2	10	0.8–1.8	
5–10	1.0–1.5	1	10	44	—[a]	Ytterskär Karlsborg
		2	7	31	1.1–1.6	
		3	7	25	1.8–2.5	
		4	7	22	1.8–2.7	
		5	7	17	1.8–2.9	
5–10	3.0–3.5	1	7	32	—[a]	
		2	5	25	0.8–1.6	
		3	5	20	1.8–2.7	
		4	5	15	2.1–3.4	
		5	5	10	2.5–4.2	
5–10	4.5–5.0	1	7	32	—[a]	
		2	7	24	—[a]	
		3	6	19	1.1–2.2	
		4	6	14	1.4–2.7	
		5	6	11	1.9–3.6	

[a] As the measurement errors were too large, a calculation of the variability value vas not possible.

In ridges the wind speed is naturally rather low. Therefore no higher values of the wind speed than 5–10 m/sec occur in the measurements made in this situation. The pilot-balloon ascents with wind speeds in the upper part of the interval 5–10 m/sec were made in the peripheral parts of the ridges and could therefore perhaps have been classified as belonging to the group "straight contour lines".

The number of difference vectors which form the base for the calculation of the wind variability is not very large. For the five-hour intervals—the longest time interval—the number is quite too small. This means that a very few *large* vectors in the distribution of difference vectors may give an undue influence to the standard vector deviation calculated from such a small distribution.

TABLE 5 *b*. *Wind variability in troughs.*

Wind speed (m/sec)	Layer (km)	Time interval (hours)	No. of synoptic situations	No. of difference vectors	Variability (m/sec) corrected for S.E.	Serial wind measurements from
< 5	1.0–1.5	1	10	130	0.8–1.1	
		2	10	102	1.4–1.6	
		3	9	81	1.6–2.0	
		4	8	66	1.9–2.3	
		5	8	37	2.3–3.1	
< 5	3.0–3.5	1	13	149	1.0–1.3	
		2	9	117	1.6–1.8	
		3	8	94	1.8–2.2	
		4	8	76	2.1–2.5	
		5	7	51	2.3–2.9	
		6	7	37	2.5–3.1	
		7	7	23	2.4–3.5	
< 5	5.0–5.5	1	10	118	1.1–1.3	
		2	7	104	1.6–1.8	Ytterskär
		3	7	83	1.6–2.0	Uppsala
		4	7	68	2.0–2.5	Gothenburg
		5	7	53	2.3–2.9	
		6	7	39	2.6–3.2	
		7	6	24	2.7–3.7	
< 5	7.0–7.5	1	9	90	0.6–0.9	
		2	7	76	0.9–1.6	
		3	5	58	1.5–2.1	
		4	5	51	1.5–2.1	
		5	5	44	1.8–2.5	
		6	5	31	1.8–2.8	
		7	5	23	2.2–3.3	
< 5	9.0–9.5	1	4	26	—[a]	
		2	4	20	—[a]	
		3	3	19	1.2–2.9	
5–10	1.5–2.0	1	11	137	1.4–1.6	
		2	10	115	2.0–2.2	
		3	10	97	2.1–2.6	
		4	10	78	2.6–2.9	
		5	9	60	2.7–3.4	
		6	8	44	3.1–3.9	
		7	5	30	2.5–3.3	

[a] As the measurement errors were too large, a calculation of the variability value was not possible.

Table 5 b (cont.)

Wind speed (m/sec)	Layer (km)	Time interval (hours)	No. of synoptic situations	No. of difference vectors	Variability (m/sec) corrected for S.E.	Serial wind measurements from
5–10	3.0–3.5	1	11	121	0.8–1.5	Ytterskär Uppsala
		2	11	100	1.7–1.9	
		3	8	80	2.3–2.7	
		4	8	63	2.9–3.6	
		5	8	50	3.0–3.9	
		6	8	37	3.2–4.0	
		7	3	24	3.2–4.4	
5–10	5.0–5.5	1	4	89	0.3–1.3	
		2	4	84	1.6–2.1	
		3	4	71	2.1–2.9	
		4	4	56	2.5–3.2	
		5	4	47	2.5–3.4	
		6	4	37	2.9–4.0	
		7	3	24	2.6–4.0	

In some cases the measurement errors were of the same magnitude as the obtained value of the standard vector deviation due to variability and errors. Consequently no wind variability values could be calculated.

In *troughs* (Table 5*b*) data were obtained from levels up to 9.0–9.5 km when the wind speed was <5 m/sec and up to 5.0–5.5 km when the wind speed was 5–10 m/sec. As in ridges, the largest wind speed in the situation mentioned here was 5–10 m/sec.

The serial pilot-balloon ascents made in troughs were rather numerous, and therefore the obtained values of the wind variability are more reliable here than when obtained in ridges. The individual series also had a duration long enough to permit a calculating of the wind variability for time intervals of, usually, seven hours.

In the 9.0–9.5 km layer, wind-variability values for time intervals of one and two hours could not be calculated because the measurement errors were too large.

Within the group *situations with straight contour lines* (Table 5c) data were obtained within wind speed intervals up to 17–20 m/sec. The number of basic vectors was, however, very different in the different wind speed intervals and in the different layers. For the 5–10 m/sec speed interval the number of basic vectors can only be considered sufficient in the 1.0–1.5 km layer. In the 10–15 m/sec interval the number of basic vectors was sufficient for all layers up to 7.0–7.5 km except for the 5.0–5.5 km layer. The values obtained for the 17–20 m/sec speed interval must be regarded as a small sample.

To some extent the gaps mentioned above are filled by means of a treatment of the synoptic radio wind measurements made twice in 24 hours at the aerological station

TABLE 5 *c*. *Wind variability in situations with straight contour lines.*

Wind speed (m/sec)	Layer (km)	Time interval (hours)	No. of synoptic situations	No. of difference vectors	Variability (m/sec) corrected for S.E.	Serial wind measurements from
5–10	1.0–1.5	1	9	61	1.1–1.6	
		2	9	49	1.9–2.6	
		3	8	40	2.5–3.3	
		4	7	30	2.7–3.5	
		5	7	20	3.1–4.6	
		6	5	10	2.7–4.6	Ytterskär Uppsala
5–10	3.0–3.5	1	4	18	0.3–1.4	Karlsborg
		2	4	16	1.5–2.7	Gothenburg
		3	4	14	1.9–3.3	
		4	4	10	2.0–3.8	
5–10	9.0–9.5	1	5	25	1.0–2.2	
		2	5	19	2.2–3.4	
10–15	1.0–1.5	1	12	125	1.9–2.3	
		2	12	105	2.7–3.1	
		3	11	80	3.0–3.6	
		4	11	62	3.5–4.4	
		5	9	43	3.9–5.0	
		6	9	23	3.2–4.5	
		7	7	11	3.5–5.8	
10–15	3.0–3.5	1	10	58	1.0–1.6	
		2	10	45	1.6–2.4	
		3	9	34	2.2–3.1	Ytterskär
		4	6	29	2.4–3.6	Uppsala
		5	6	21	2.4–3.6	Karlsborg
10–15	5.0–5.5	1	5	21	1.1–2.3	
		2	5	16	3.1–4.8	
		3	4	11	3.4–5.8	
10–15	7.0–7.5	1	5	42	1.6–2.1	
		2	5	32	2.2–3.1	
		3	5	29	2.6–3.7	
		4	3	20	2.8–4.0	
		5	3	15	2.9–5.4	
17–20	1.0–1.5	1	2	24	1.9–2.8	
		2	2	20	2.8–4.1	Uppsala
		3	2	16	2.8–4.3	Karlsborg
		4	2	12	3.2–5.0	

TABLE 5 *d*. *Wind variation values obtained from upper-wind measurements made at Bromma. Time interval 12 hours.*

Wind speed (m/sec)	Altitude (km)	No. of difference vectors	Wind variation (m/sec) corrected for S.E.	Average of wind-variation values from levels 3, 5 and 9 km
5–10	3	299	4.9– 5.3	
	5	284	6.2– 6.8	6.2–7.0
	9	103	7.4– 8.8	6.6
	12	101	3.4– 4.1	
10–15	3	202	6.0– 6.6	
	5	223	7.2– 8.1	7.0–8.1
	9	71	7.9– 9.6	7.6
	12	82	4.4– 5.7	
15–20	3	120	7.8– 9.0	
	5	162	7.1– 8.2	8.3–9.5
	9	89	9.4–11.4	8.9
	12	56	3.3– 6.2	
20–25	3	39	7.0– 9.1	
	5	70	8.2– 9.5	7.2–9.4
	9	62	6.5– 9.7	8.3
	12	23	—[a]	
25–30	3	—	—	
	5	44	6.1– 9.0	6.1–9.0
	9	34	—[a]	7.6
	12	—	—	

[a] As the measurement errors were too large, a calculation of the variability value vas not possible.

at Bromma for the years 1951–1953. From existing chronological lists of the measurements made at Bromma giving wind values at the altitudes 3, 5, 9 and 12 km, difference vectors between wind values with a time interval of 12 hours were calculated.[1] With this large time difference between the measurements it was of course not possible to obtain a sequence of wind values in a quasi-stationary wind field with a duration long enough to permit a computation of the variability. However, only sequences of winds with nearly the same direction and speeds within any of the intervals 5–10, 10–15, 15–20, 20–25 and 25–30 m/sec were treated. Therefore it was to some extent probable that the synoptic situation had not changed rapidly during

[1] In reality the wind values did not relate exactly to the given altitudes but to layers centered about them. The thickness of these layers depended on the rate of ascent of the radiosonde balloons. As a rule it was about 400 m/min.

the wind sequences. Obtained values of the standard vector deviations are shown in Table 5*d*. In view of the way in which they are obtained we prefer to call them *wind variations* with time.

Table 5*d* shows that the wind variation values within the different wind-speed intervals increase somewhat with altitude, up to and including the level of 9 km. This may, of course, indicate that the wind variation is dependent on the altitude, but it may also be due to the uncertainty in the determination of the measurement errors.

The considerable diminution in the wind variation from the 9 km level to the 12 km level is striking. This indicates that the wind variation in the stratosphere is appreciatby less than in the troposphere. This has also been pointed out by Muench (1958), who studied the wind variation with time for the altitudes 16, 20 and 25 km for midseasonal months and for the American continent between 30°N and 60°N. Muench indicated that the wind variations with time (24–48 hours) of stratospheric winds are much less than for tropospheric winds.

In the measurements from Bromma also some wind-variation values could not be calculated because the measurement errors were too large.

4.3 Influence on the wind variability from altitude, synoptic situation and wind speed

In our discussion of previous investigations of upper-wind variations (section 1 above), it appeared that only the influence of altitude upon the variations had been discussed by the different authors. The influence of, for instance, the synoptic situation and the wind speed has not been treated. It therefore seems suitable to make some comments on this here.

(a) *Wind variability and altitude*

In order to obtain a picture of the variability at different altitudes, the variability values of Tables 5*a–c* are graphically represented in Fig. 14.

For our three synoptic situations and for different speed intervals the variability of each layer is visualized by limited horizontal straight lines. Fig. 14 shows that there is no significant reason why the variability should not be constant with the altitude in the layer from 1 to 9.5 km, other circumstances being unchanged.

(b) *Wind variability and synoptic situation*

Fig. 15 gives a comparison between the wind variability in the three types of synoptic situations. As seen above the wind variability was not influenced by the altitude as such. Therefore the means of the variability values of all layers were calculated and represented by the limited vertical straight lines in Fig. 15. Ridges were

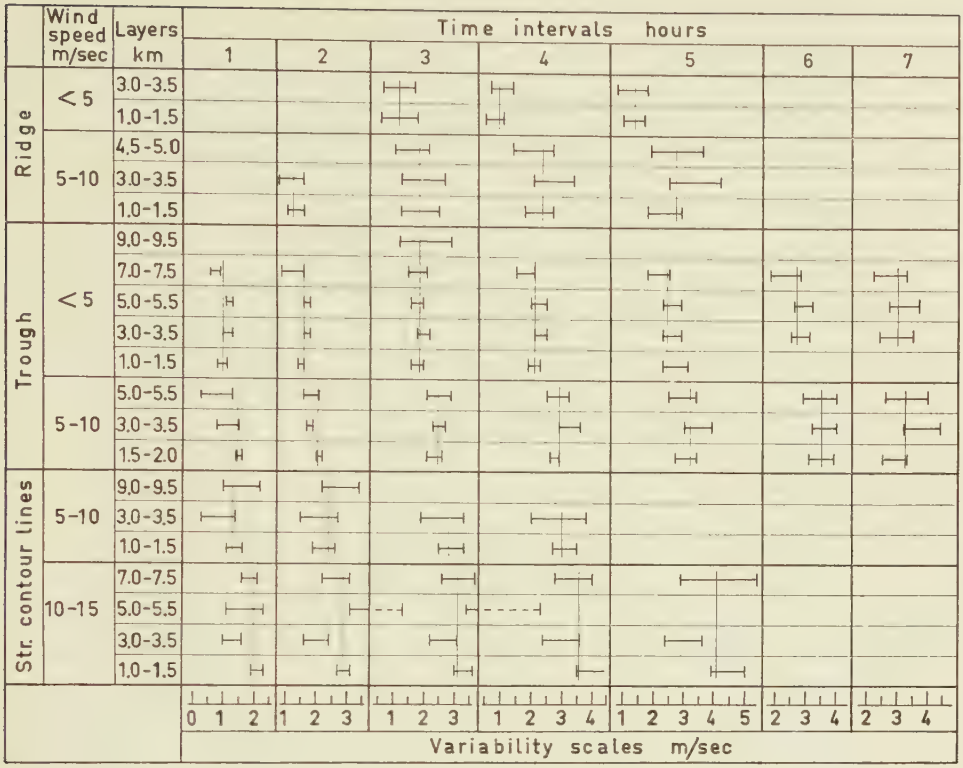

Fig. 14. Wind variability and altitude.

represented by the capital letter R, troughs by T and situations with straight contour lines by S.

Only the two wind-speed intervals <5 m/sec and 5–10 m/sec are included in Fig. 15, as they are the only intervals common to at least two types of situations. As far as the type ridge was concerned, the wind variability was clearly less in ridges than in troughs when the wind speed was <5 m/sec. When the wind speed was 5–10 m/sec there was no marked difference between the three synoptic situations.

That the difference between the variability values from the different situations should diminish with increasing wind speed is rather natural. In the low speed interval the ridge or the trough line was situated near the measuring place but in the higher speed interval this line was beside the measuring place. We then had a case more similar to the situations with straight contour lines.

Therefore, in practical applications, where the wind variability is of importance, it does not seem necessary to consider the synoptic situation, except when a ridge occurs and the wind speed is lower than 5 m/sec.

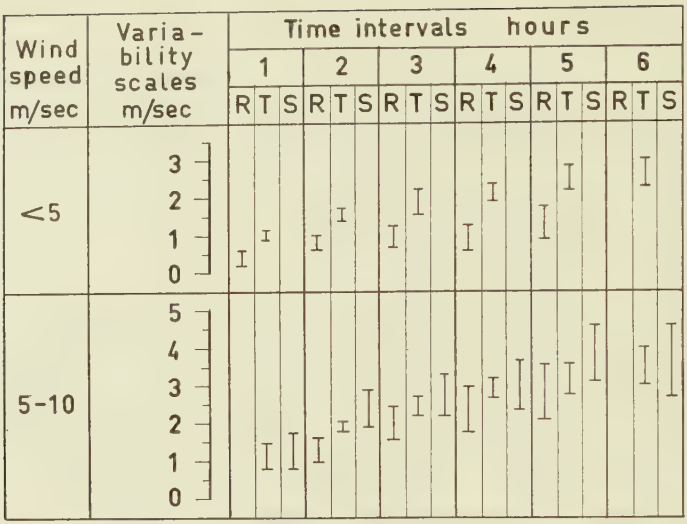

FIG. 15. Wind variability and synoptic situation. *R* indicates ridges, *T* troughs, and *S* situations with straight contour lines.

(c) *Wind variability and wind speed*

It appears from Fig. 15 that the variability increases somewhat with increasing wind speed. This is more evident in Fig. 16, which shows the variability as a function of the wind speed. Fig. 16 has two parts, one calculated for the situation ridge and one common to the two other types of situations. The variability values are means of all layers. The letters *a–d* indicate the wind speed at which $a < 5$, $b = 5$–10, $c = 10$–15, and $d = 17$–20 m/sec.

5. A connection between wind variability and time interval

5.1 Theoretical considerations

The variability values are in some cases obtained from so very few measurements that it would be valuable to have a model to which the variability values could be fitted. Then it also ought to be possible to calculate the variability for other time intervals than these which were obtained from the measurements.

Such a model can be obtained from Taylor (1921). Taylor deduced the expression

$$[X^2] = 2\,[u^2] \int_0^T \int_0^t R_\xi \, d\xi \, dt,$$

which—in the case of uniformly distributed turbulence—describes the diffusion of smoke particles, colouring matter or other properties which adhere to each particle of a fluid during its motion.

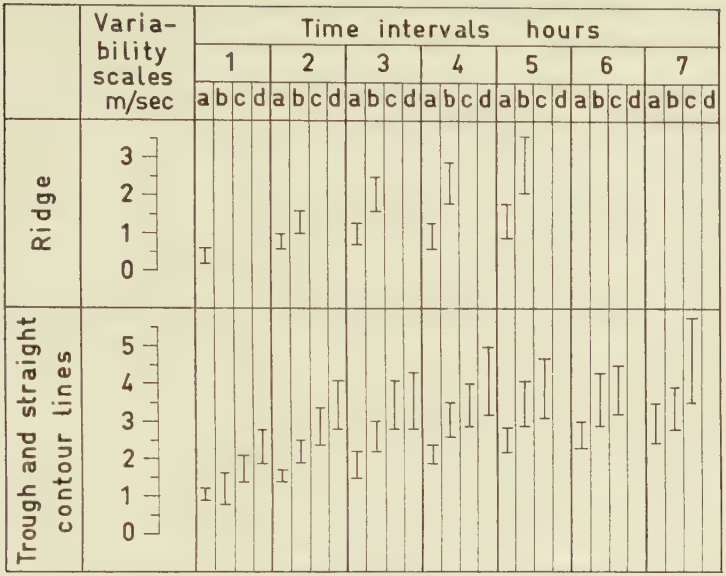

FIG. 16. Wind variability and wind speed. The letters *a–d* in the time-interval columns indicate the wind speed at which $a < 5$, $b = 5$–10, $c = 10$–15, and $d = 17$–20 m/sec.

$[X^2]$ is the square of the standard deviation of the distance X traversed by a particle in time T,

$[u^2]$ is the square of the standard deviation of the velocity u of a particle,

R_ξ is the correlation coefficient between the value of u for a particle at a time t and the value of u for the same particle after an interval of time ξ.

Two cases can be considered. First suppose that T is so small that R_ξ does not differ appreciably from 1 during the interval T. In this case

$$\int_0^T \int_0^t R_\xi \, d\xi \, dt = \tfrac{1}{2} T^2$$

and

$$\sqrt{[X^2]} = T \sqrt{[u^2]}.$$

That is to say, the standard deviation of a particle from its initial position is proportional to T, when T is small.

In the second case it is supposed that R_ξ would fall to zero for large values of ξ.

Then

$$\int_0^T \int_0^t R_\xi \, d\xi \, dt = \int_0^T \left[\int_0^t R_\xi \, d\xi \right] dt.$$

TABLE 6. *Averages of the wind-variability values (m/sec) from all layers.*

Wind speed (m/sec)	Synoptic situation	Time interval in hours 1	2	3	4	5	6	7
< 5	Ridge	0.2–0.6 0.4	0.6–1.0 0.8	0.7–1.3 1.0	0.6–1.3 1.0	0.9–1.8 1.4	—	—
< 5	Trough	0.9–1.2 1.1	1.4–1.7 1.6	1.5–2.2 1.9	1.9–2.4 2.2	2.2–2.9 2.6	2.3–3.0 2.7	2.4–3.5 3.0
5–10	All situations	0.9–1.6 1.3	1.8–2.5 2.2	2.0–3.0 2.5	2.4–3.6 3.0	2.9–4.1 3.5	2.9–4.3 3.6	2.8–3.9 3.4
10–15	Straight contour lines	1.4–2.1 1.8	2.4–3.4 2.9	2.8–4.1 3.5	2.9–4.0 3.5	3.1–4.7 3.9	3.2–4.5 3.9	3.5–5.8 4.7
17–20	Straight contour lines	1.9–2.8 2.4	2.8–4.1 3.5	2.8–4.3 3.6	3.2–5.0 4.1	—	—	—

Moreover it is assumed that

$$\lim_{t\to\infty} \int_0^t R_\xi \, d\xi$$

is finite and equal to I.

Then

$$\int_0^T \left[\int_0^t R_\xi \, d\xi \right] dt = \int_0^T I \, dt = IT$$

and

$$\sqrt{[X^2]} = T^{\frac{1}{2}} \sqrt{2 I [u^2]}.$$

That is to say, the standard deviation of X is proportional to $\sqrt{T}$.

The expressions for $R_\xi = 1$ and $R_\xi = 0$ can be combined in the form

$$\sqrt{[X^2]} = k_1 \cdot t^p, \tag{13}$$

where k_1 is a function of $[u^2]$ and $\frac{1}{2} < p < 1$.

According to the definition above, $\sqrt{[X^2]}$ is the standard deviation of the distance X traversed by a particle in time T. Taylor considered air particles, but the nature of the particles is not essential. The expression (13) is also valid for the spreading of pilot balloons. In our case the spreading of the balloons in a series of ascents is measured by the standard vector deviation of the differences between consecutive wind vectors. But this deviation is, according to our definition, equal to the wind variability.

Consequently, the wind variability s is proportional to $\sqrt{[X^2]}$, and therefore

$$s = k \cdot t^p. \tag{14}$$

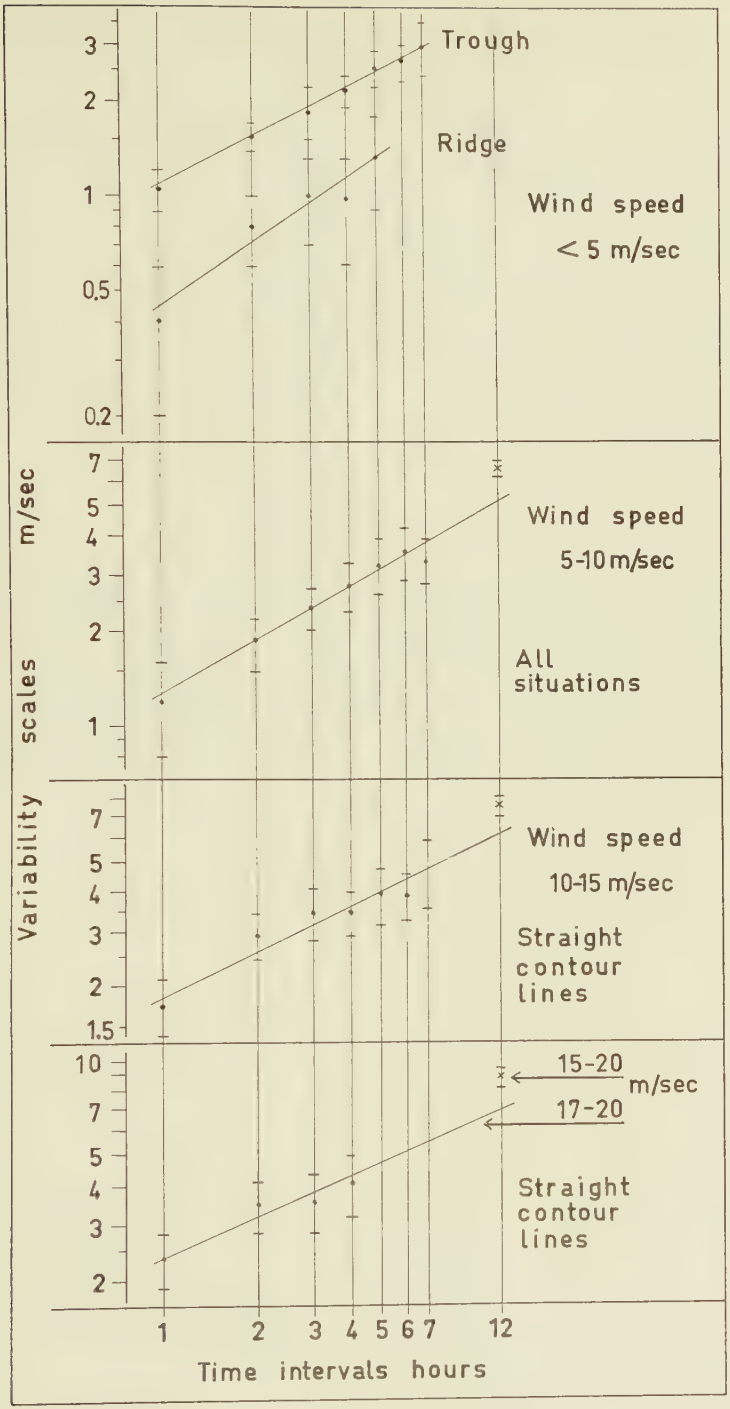

FIG. 17. Wind variability values in loglog diagram. The plots marked with x (on the 12-hour line only) were obtained from measurements at Bromma.

5.2 A check of the expression $s = k \cdot t^p$ against observed variability values

In a loglog diagram the expression $s = k \cdot t^p$ is represented by a straight line. Therefore it is very suitable as a model for the connection between wind variability and the length of the time interval. It is also easy to check against observed values. In order to make the check, the values from Table 6 were used. They were plotted in the loglog diagram of Fig. 17 and straight lines were fitted to the plotted points.

The wind-variation values (time interval of 12 hours) from the measurements at Bromma (Table 5*d*) were also plotted in the diagram of Fig. 17. These values are somewhat too high in comparison with the values obtained from Table 6. However, this is rather natural, since the values from Bromma only to some degree fulfil the condition to be obtained from measurements made in a quasi-stationary wind field.

5.3 Calculation of the values of p and k

Consider first the value of p. Theoretically $\frac{1}{2} < p < 1$, where the value $\frac{1}{2}$ is valid for large time intervals. It seems legitimate to assume that in the expression (14) one hour may be regarded as a large time interval. The value of p may therefore be put equal to $\frac{1}{2}$ in the present case of wind variability.

The value of k is equal to the value of s, when $t = 1$. Since our pilot-balloon ascents were made with time intervals of one hour, the unit for t is one hour. Therefore k is equal to the wind variability value for one hour. From the values of Table 6 the values of p and k were calculated by the method of the least squares. Table 7 gives the result.

TABLE 7. *Values of p and k for different wind-speed intervals.*

Wind speed (m/sec)	Synoptic situation	p	k
< 5	Ridge	0.67	0.3–0.6 0.5
< 5	Trough	0.50	0.9–1.2 1.1
5–10	All situations	0.53	1.1–1.7 1.4
10–15	Straight contour lines	0.46	1.6–2.3 2.0
17–20	Straight contour lines	0.37	2.0–2.8 2.4
	Average	0.51	

TABLE 8. *Values of k estimated from measurements made at Bromma.*

Wind speed (m/sec)	Estimated k-values
5–10	1.8–2.0
	1.9
10–15	2.0–2.3
	2.2
15–20	2.4–2.7
	2.6
20–25	2.1–2.7
	2.4
25–30	1.8–2.6
	2.2

Among the values of p and k the first values, 0.67 and 0.3–0.6 respectively, and the last values, 0.37 and 2.0–2.8, are obtained from relatively few basic data. It seems therefore that the largest weight ought to be applied at the three middle values. However, the average value of p, 0.51, shows a very good agreement with the theoretical value given above.

Concerning the measurements made at Bromma it is probable that the expression $s = k \cdot t^p$ only is valid to some extent, since these measurements were made in wind fields which cannot be considered as quasi-stationary. It is, however, not probable

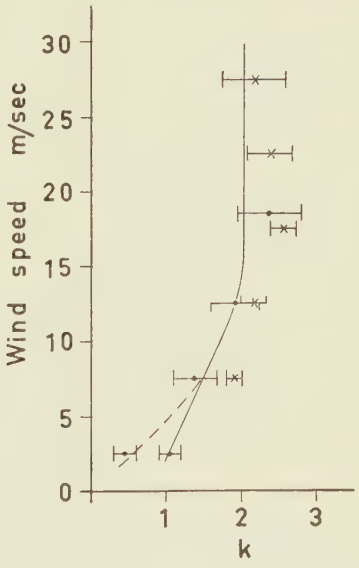

FIG. 18. Values of k as a function of the wind speed. The dashed portion of the curve refers to the variability in ridges. ● Measurements with radar. × Measurements at Bromma. The measurements marked with × are assumed to be somewhat too large (see page 52).

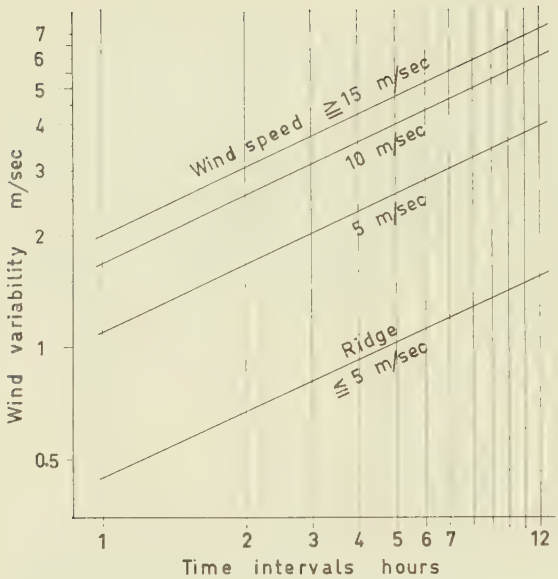

Fig. 19. Wind-variability diagram. Valid for the troposphere above the frictional layer.

that the changes were large (see section 4.2 above and Fig. 17). Therefore, and in the absence of anything better, the expression $s = k \cdot t^p$ has been used as an approximation to the real connection between the wind-variation values obtained from the Bromma measurements and the time intervals. In this case it is impossible to calculate the values of p and k by the method used above, as there is only one time interval (12 hours) within each of the wind-speed classes. Therefore we have made an estimation of the values of k using a p-value equal to 0.5. This value of p is too small and so obtained values of k are somewhat too large. Table 8 gives the obtained values of k.

With the k-values from the Tables 7 and 8 the curve in Fig. 18, which gives k as a function of the wind speed, was constructed. The dashed portion of the curve refers to the variability in ridges. When drawing the curve consideration was taken of the fact that the k-values from Table 8 were somewhat too large. It is evident from Fig. 18 that *the variability increases with increasing wind speed, but only to a certain limit at about 15 m/sec.*

With the k-values from Fig. 18 and assuming $p = 0.5$, the diagram in Fig. 19 was constructed. This diagram is intended to be a "variability diagram" from which one can obtain the wind variability for different wind speeds and time intervals for winds aloft within the troposphere above the frictional layer.

6. Some applications of the wind variability

Information on the magnitude of the variability is of value in experimental tests, for instance, in order to ascertain the corrections necessary for an efficient use of

artillery firings. These tests are normally made in synoptic situations during which the wind variations can be supposed small. Therefore a wind-variation parameter calculated from data obtained by measurements made during not quasi-stationary situations is unnecessarily large.

Decisions on what length of time a measurement is representative of the real wind conditions and how accurate instruments should be depend among other things also on the variability.

As mentioned in the introduction, the usual method of determining the winds aloft does not permit simultaneous measurement and use of the wind data. There are several reasons for this. The pilot balloon must pass through the entire altitude interval of interest, the readings must be converted into wind values, these values must be delivered to the consumer, who must often make some preparations after having got the wind values, but before the activity for which the wind values are required can start. In all, it does not seem pessimistic to consider one hour as the minimum time interval between the measurement and the use of the wind values obtained. Often an upper-wind measurement cannot be especially made for a certain activity and, therefore, perhaps a rather old measurement must be used. Consequently, according to the wind variability, it is not practical to expect the wind values obtained to coincide exactly with the real wind at the time of use. However, the difference to be expected in a situation with a quasi-stationary wind field (the unavoidable part of the difference) can be estimated from the variability diagram in Fig. 19.

For many purposes it is necessary to use upper-wind forecasts instead of direct measurements. In such cases it would be valuable if the wind data given in the forecast were supplemented with some specifications as to the variability.

The variability is also of importance in connection with problems concerning the accuracy of upper-wind reports. Sometimes, for example, a consumer's possibly unreasonable demands on accuracy may be refuted by reference to the variability.

The variability diagram also may be of value in the analysis of upper-air maps. These maps will normally contain a non-apparent error, since the plotted winds will have been derived from pilot-balloon ascents which were not all made simultaneously.

The variability diagram also has an application in the meteorological prediction of radioactive close-in fallout, i.e. the contaminated radioactive material which has been explosively distributed in the atmosphere by the burst of a nuclear weapon and, when acted upon by gravity and winds aloft, drops to the earth's surface in the vicinity of the site of the burst. The prediction is based on the basic ballistic equation, which applies to any object falling through the atmosphere and affected by the wind at all levels and, in component form, can be written as follows:

$$x = \sum \frac{V_x(h)}{w(r, h)} \Delta h, \qquad y = \sum \frac{V_x(h)}{w(r, h)} \Delta h,$$

where x and y are the distances travelled in the component directions; $V_x(h)$ and $V_y(h)$ are the wind velocity components at a given altitude h; $w(r, h)$ is the vertical-fall velocity applicable to a given object of known size, shape, and density existing at a given altitude h; and Δh is the altitude increment in which $w(r, h)$, $V_x(h)$ and $V_y(h)$ are considered to remain constant. With known values of the variables a fallout plot (see Fig. 2) can be constructed. The wind values are derived from upper-wind measurements, upper-air maps or predicted wind and normally the wind values are assumed to remain constant with time. The wind at any level, however, will not remain constant with time and so an error in the fallout area is obtained. The smallest correction to be expected (when a quasi-stationary wind field occurs or when the wind values are obtained from a forecast and the values are correctly predicted) can be derived from the variability diagram.

Part 2

UPPER-WIND STRUCTURE IN FRONTAL LAYERS AND INVERSIONS

1. Introduction

In the preceding part we have dealt with the variability of upper wind, defined as the variation of the wind in a quasi-stationary wind field. It was assumed that the variation of the wind in a changing wind field was fairly well treated when using different prognostical methods. However, the structure of the wind aloft in connection with frontal layers is not satisfactorily known. Our knowledge is for the most part obtained from synoptic wind measurements made at places situated at great distances from each other or, indirectly, by studying the geostrophic wind and the temperature field (see, for instance, Palmén (1948*a*). As the geostrophic wind can deviate appreciably from the real wind, it would be of value to investigate the wind structure in connection with frontal layers by means of upper-wind measurements adjacent both in time and space. As far as I know, such series of wind measurements have not been made. I only have knowledge of one study specially treating the wind structure in a frontal layer by means of pilot-balloon ascents, viz. a study by Reed (1957). Reed used, however, only one pilot-balloon ascent made in connection with a simultaneous radio-sounding.

In the material put at the disposal of the present author for investigating the variability of the upper wind, however, there were some serial pilot-balloon ascents, which were made in connection with fronts. Serial wind measurements in connection with a frontal passage in January 1960 were also available. The material was rather scanty, but from it some examples of the wind structure in connection with frontal layers could be obtained. Of course, it would have been very valuable if serial ascents of radiosondes could also have been made on the same occasion as the wind measurements. In this way comparisons between temperature cross-sections and wind cross-sections could have been made. In want of such soundings, ordinary synoptical radio-soundings had to be used in order to define the frontal layers. Of course, it was not then possible to obtain the exact positions of the frontal boundaries. However, using a three-dimensional analysis, the most probable positions of the frontal boundaries could be obtained. This three-dimensional analysis included a study of tempograms as well as upper-air maps.

A series of pilot-balloon ascents made in a situation with subsidence inversions were also treated to serve as an example of the upper-wind conditions in connection with inversion layers.

2. Visualization of the wind conditions

In the examples given below the wind conditions were visualized by means of isopleth diagrams, one for the direction and one for the speed in each example (see, for instance, Figs. 25 and 26). The isopleths were constructed mainly from radar wind measurements, but in some cases these measurements were supplemented with radio wind measurements. Obtained direction and speed values were plotted in a height-time diagram, and the isopleths were drawn freehand and somewhat smoothed. The isopleths give direction and speed of the wind at a given time as a function of height or, at a given height, as a function of time. The wind-direction isopleths (isogones) give the wind direction in degrees from the north over east, and the wind-speed isopleths (isovels) give the speed in m/sec. The height is given in km.

Every pilot-balloon ascent was represented in the diagrams by a straight line, limited by a dot at the lower and upper ends, indicating the vertical extent of the wind measurement and also its location in the time scale. The lines were inclined in the direction of the time scale, as a value from a high level was obtained later than a value from a lower level. Since pilot balloons drift with the wind, the wind values obtained from a series of ascents cannot be attributed to the exact vertical plane through the place of measurement. The isopleth cross-sections must therefore be regarded as rather thick vertical layers.

It is, of course, not directly clear from the isopleth diagrams that the wind singularities in our examples really were connected with frontal layers. It could, however, be shown from composite maps that a front really had passed the place of ascent at such a time that the pilot balloons must have passed through the frontal layer. The frontal layers could also be determined by a three-dimensional analysis without the help of the serial wind measurements. As the frontal layers thus determined coincided with the wind singularities in the isopleth diagrams, these singularities must—according to the thermal wind equation—be caused by the frontal layer.

In analysing the maps used for our discussion of the meteorological conditions, consideration was taken to the analyses made by the Weather Bureaux at the Meteorological and Hydrological Institute of Sweden, the Swedish Military Weather Service and Täglicher Wetterbericht des Deutschen Wetterdienstes, i.e. as the analyses have appeared on working maps and in published bulletins.

Serial pilot-balloon ascents were made at Ursvik (59°23′N, 17°57′E), Ytterskär (59°58′N, 18°52′E) and at Gothenburg (57°44′N, 11°57′E). For constructing the vertical cross-sections through the frontal layers and for our discussions of the upper-air conditions, aerological ascents from the following stations were used. Bromma (59°21′N, 17°57′E), Torslanda (57°43′N, 11°47′E), Karlsborg (58°31′N, 14°30′E), Jokioinen (60°49′N, 23°29′E), Sortavala (61°43′N, 30°43′E), Petrozavodsk (61°48′N, 34°16′E) and Copenhagen (55°38′N, 12°40′E) (see Fig. 3).

All times given in the examples below are Swedish Standard Time.

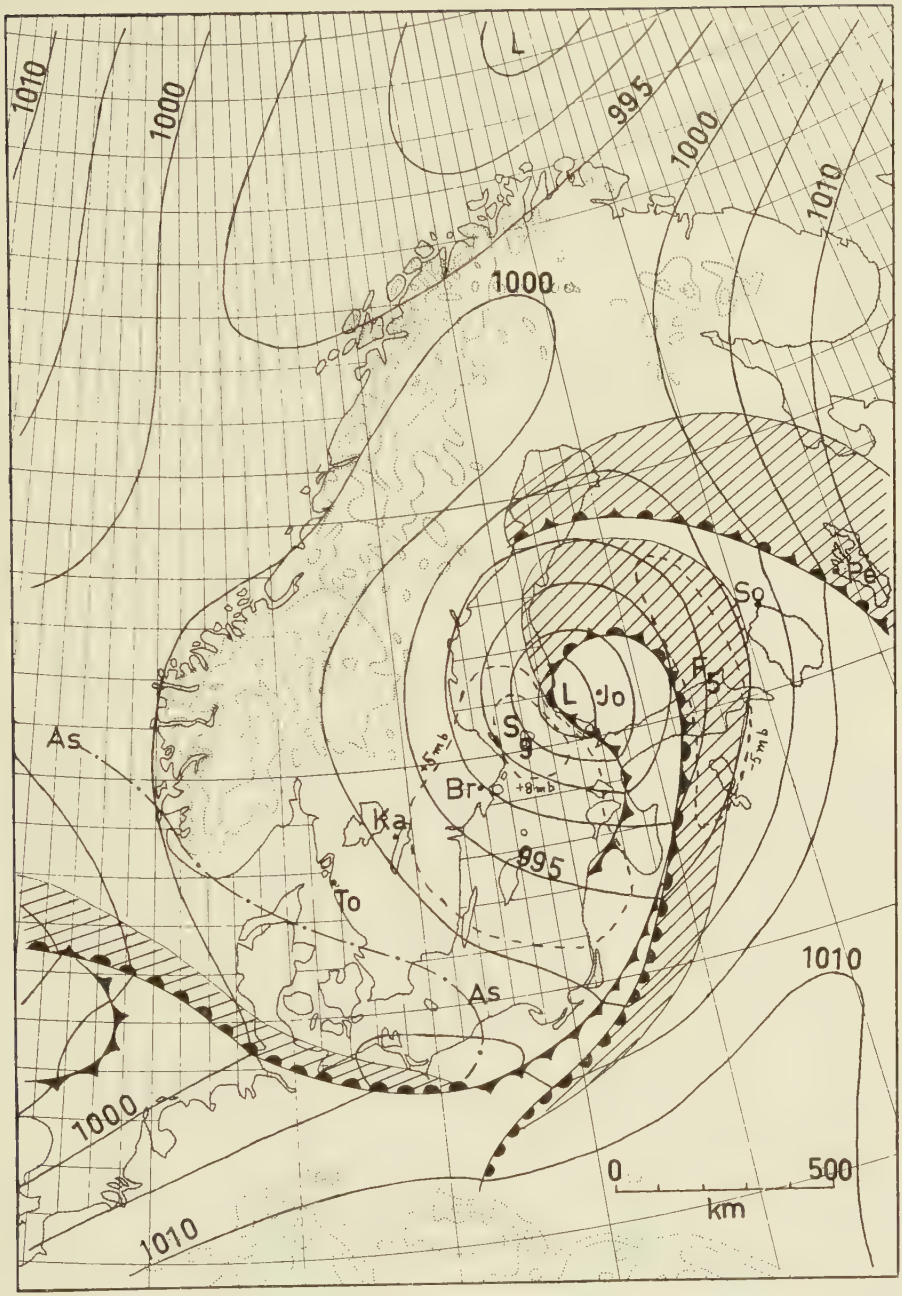

FIG. 20. Composite map, 1300 January 19, 1960. — · — Limit for altostratus.

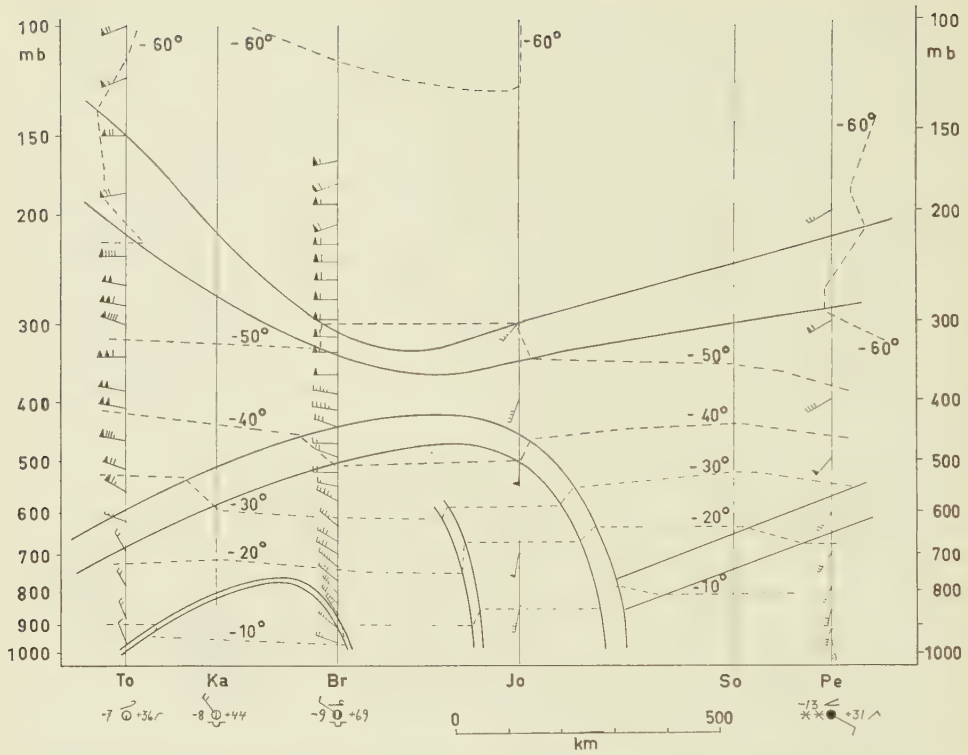

Fig. 21. Vertical cross-section from Torslanda to Petrozavodsk, 1300 January 19, 1960. Dashed lines are isotherms (°C), heavy lines frontal boundaries, tropopaus, and inversion. Observed winds are plotted in the conventional manner. Full barb 5 m/sec, half barb 2.5 m/sec, triangular barb 25 m/sec.

3. Examples

3.1 Example 1

Warm front and back-bent occlusion.

Serial wind measurements from Ursvik, January 19, 1960, 09^h–15^h.

(a) *Synoptic situation*

During the night of January 18–19 an Arctic front cyclone passed Svealand in a north-easterly direction. During this movement the cyclone rapidly began to occlude and to deepen. In agreement with the general model of such a cyclone, it developed a back-bent occlusion. The wind measurements were made during the passage of this back-bent occlusion and also through the warm front layer belonging to the next cyclone in this cyclone series. The composite map at 1300 (Fig. 20) shows the frontal system.

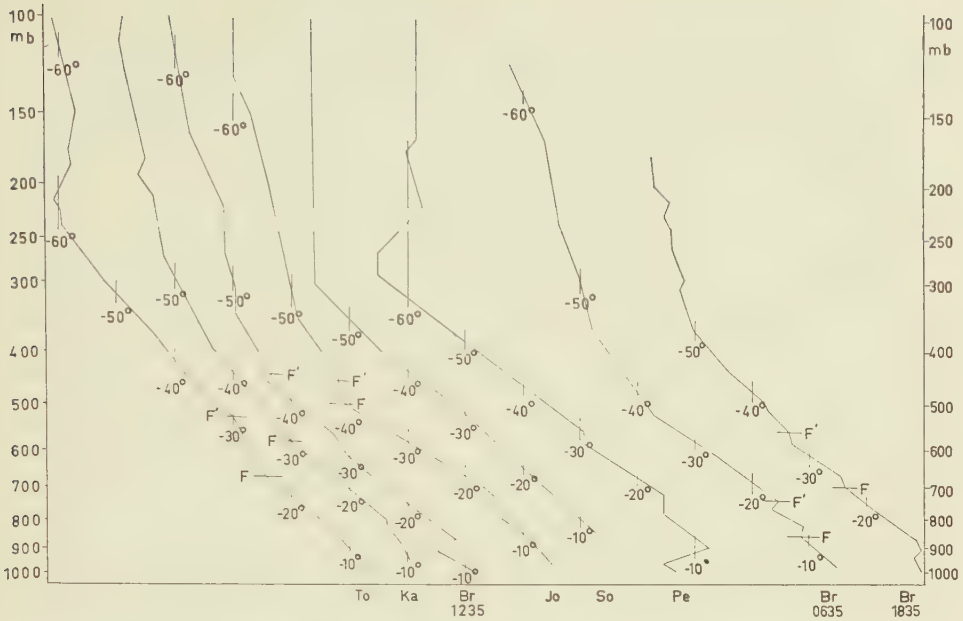

FIG. 22. Tempograms from Torslanda, Karlsborg, Jokioinen, Sortavala and Petrozavodsk stations at 1300 and Bromma at 0635, 1235 and 1835, January 19, 1960.

The upper-air conditions are shown by the vertical cross-section of Fig. 21. Since the serial pilot-balloon ascents were made at Ursvik the most interesting part of the frontal system in this example is in a vertical cross-section passing through Ursvik and normal to the fronts at the earth's surface. Therefore the aerological ascents from Torslanda, Karlsborg, Bromma, Jokioinen, Sortavala and Petrozavodsk (on the map of Fig. 20 marked with To, Ka, Br, Jo, So and Pe) were used to construct the section. Since Karlsborg only was represented at 1300 and the serial wind measurements were centered around this time the cross-section was constructed from observations at 1300. The tempograms used are shown by Fig. 22.

The vertical cross-section was constructed in agreement with the occlusion model introduced by Bergeron (1936). The fronts at the earth's surface were determined in the section by means of the composite map. The inclination of the cold front layer between Jokioinen and Sortavala was estimated in the lower levels by means of the 700 mb map (Fig. 23). At the 500 mb level (Fig. 24) the coldest air was situated westward from Jokioinen. Therefore the nearly isothermal layer between 500 mb and 450 mb over Jokioinen (*FF'*, Fig. 22) must be connected with the cold front layer. From these facts the cold front layer was settled.

The fact that (below the 500 mb level) the temperatures over Bromma were much lower than those over Jokioinen at corresponding levels supports the existence of

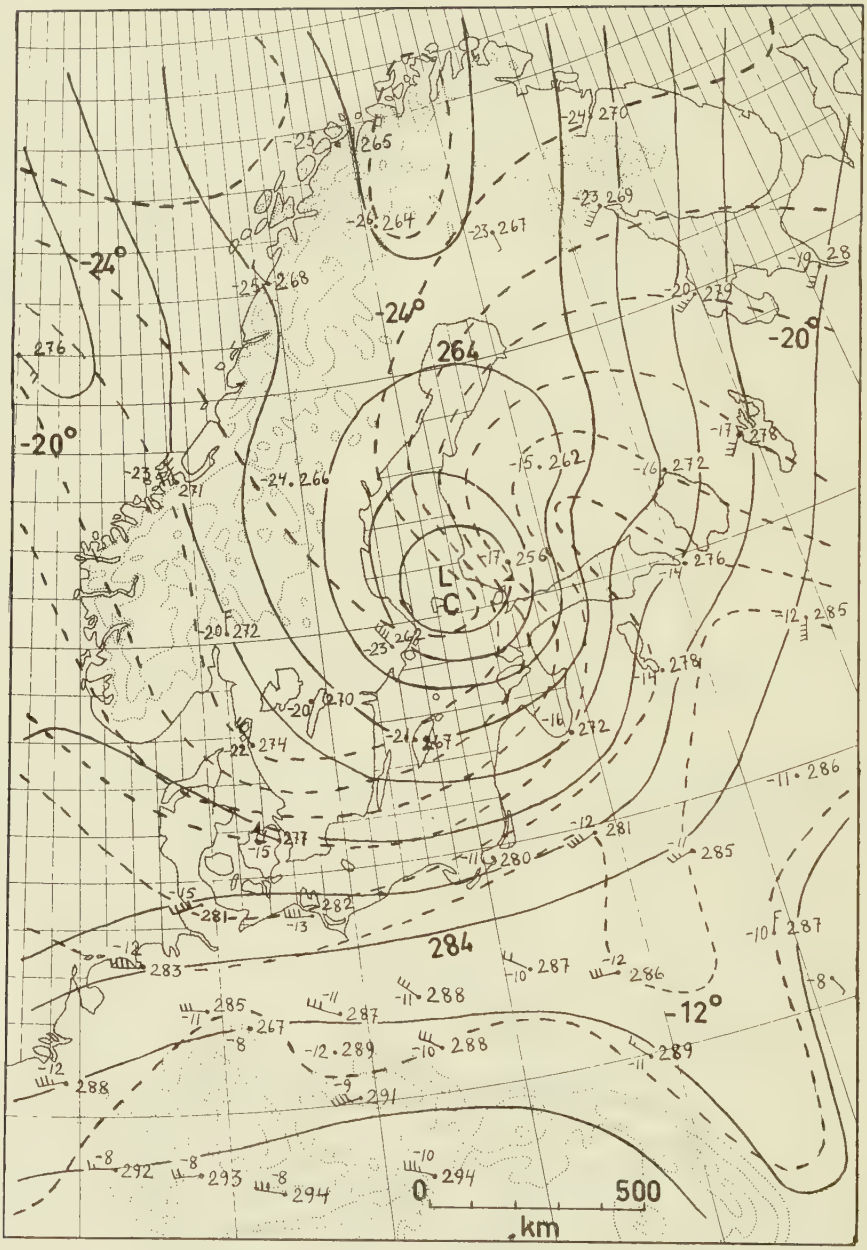

FIG. 23. 700 mb map. 1300 January 19, 1960. Solid lines are isohypses, drawn for every 4 geopotential decameter; dashed lines are isotherms for every 2°C.

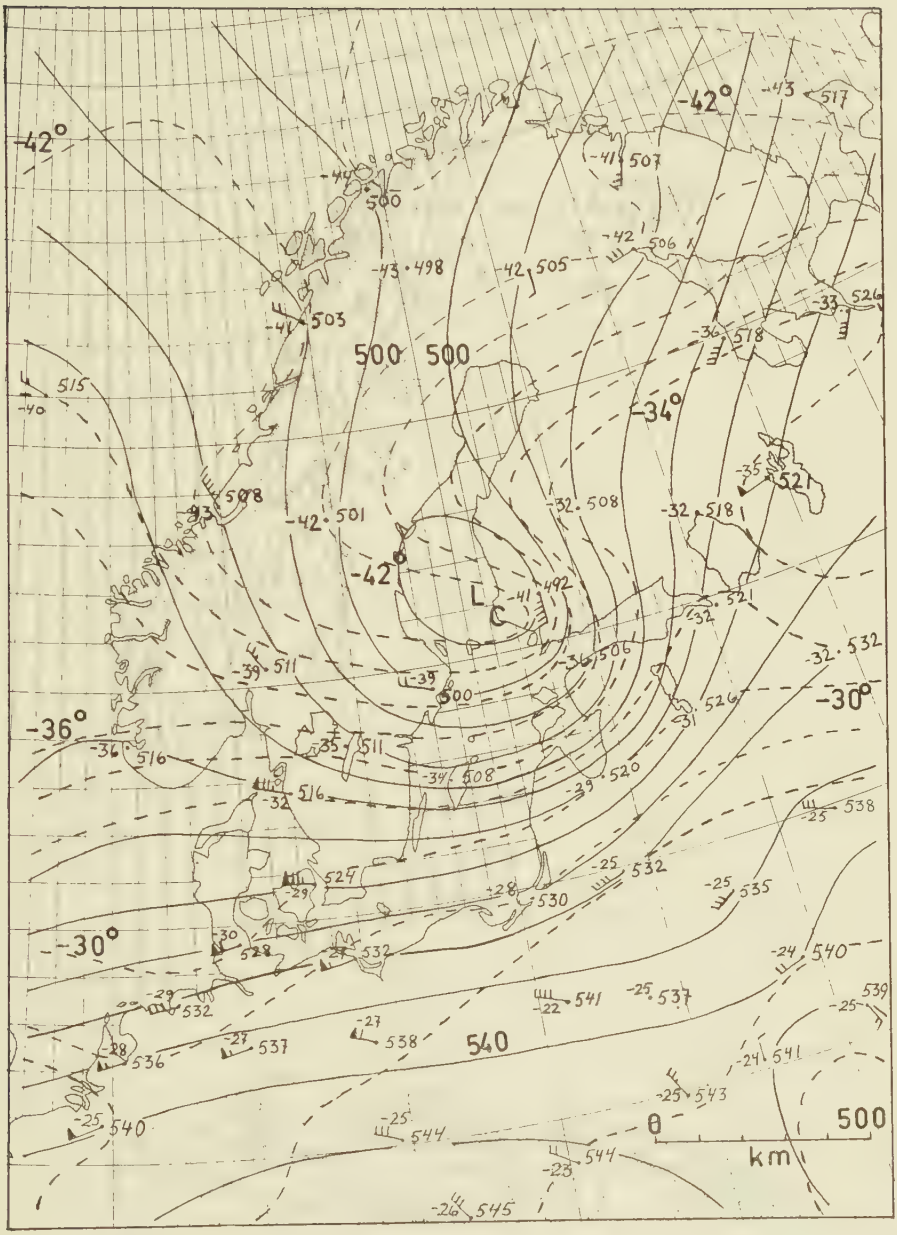

FIG. 24. 500 mb map, 1300 January 19, 1960. Solid lines are isohypses, drawn for every 4 geopotential decameter; dashed lines are isotherms for every 2°C.

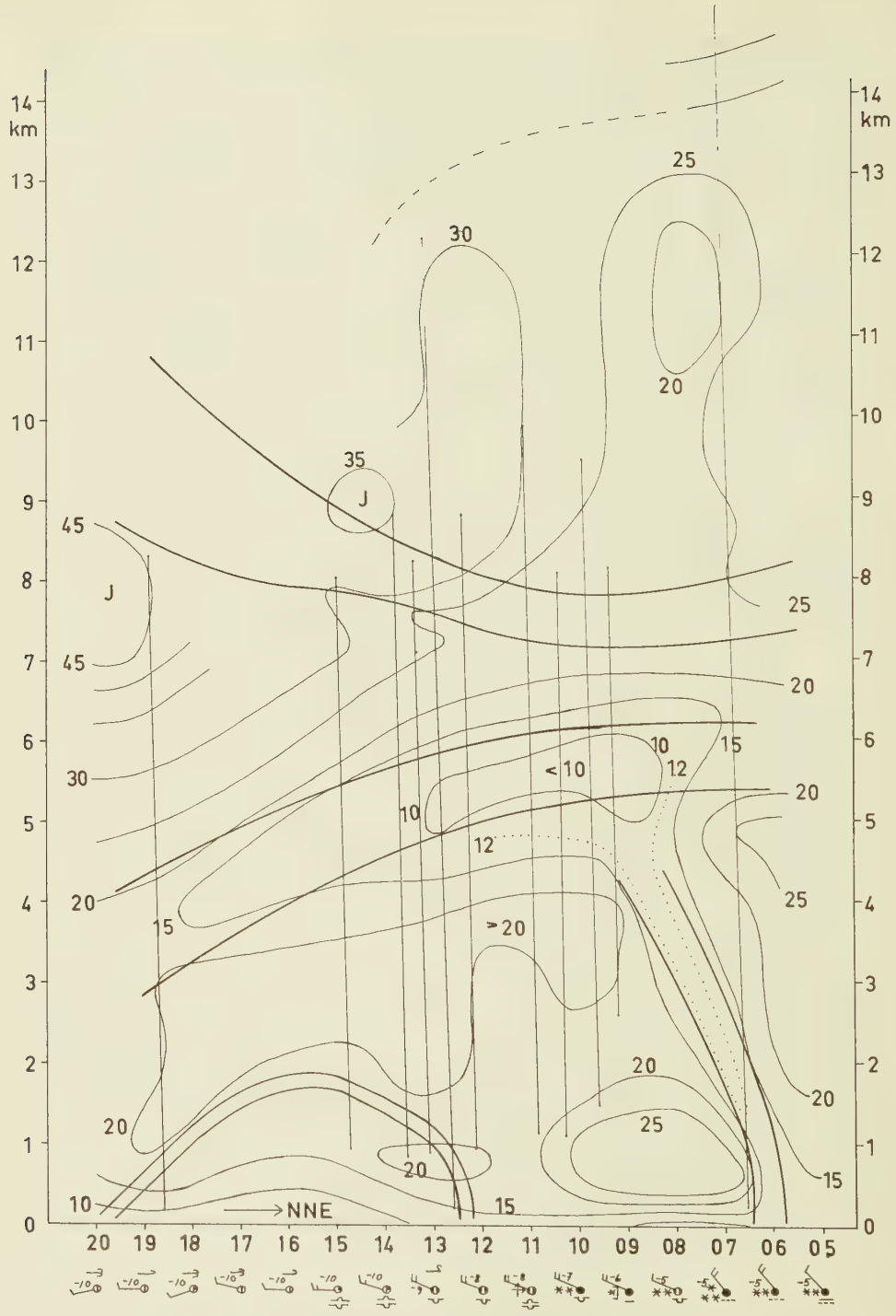

FIG. 25. Isovel diagram, Ursvik, January 19, 1960. Thin solid lines are isovels for every 5 m/sec (*observed* wind), dotted line intermediate value, heavy lines frontal boundaries, tropopaus and inversion. Station plots from Bromma.

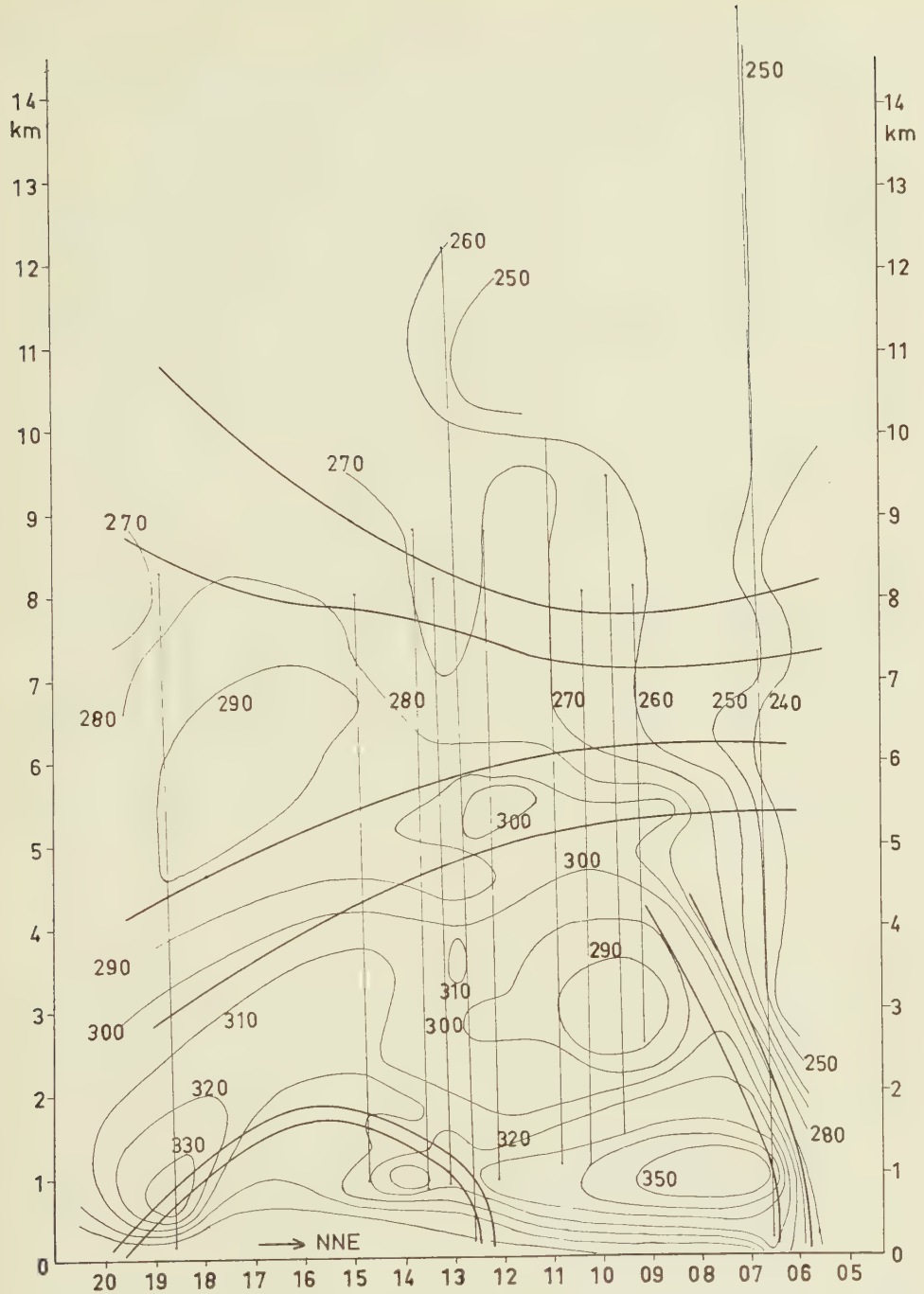

FIG. 26. Isogon diagram, Ursvik, January 19, 1960. Thin solid lines are isogons (degrees from the north) for *observed* wind, heavy lines are frontal boundaries, tropopaus and inversion.

the back-bent occlusion. The temperature difference, however, ceased at about the level of 540 mb, indicating that the back-bent occlusion layer did not exist over this level. The inclination of the back-bent occlusion layer was estimated from the 700 mb map.

The warm front at the earth's surface belonging to the next cyclone in the series was at 1300 about 450 km south-westward from Torslanda. According to the cloud types reported by the stations in Denmark and in the southern part of Sweden (the limit for altostratus is indicated in Fig. 20; see also Fig. 25 where the station plots after 13^h indicate that Cirrostratus and Cirrus occur over Bromma), the warm front layer must extend over this region. Assuming a slope of 1:100 the frontal layer must at Torslanda be situated at the level of about 580 mb at the most. The 700 mb map shows that the frontal layer was situated to the south-west of Torslanda. Consequently, over Torslanda the frontal layer must be situated higher than the 700 mb level. The tempogram from this station shows a layer with weaker lapse rate between 670 and 520 mb (FF', Fig. 22). It is therefore probable that this weaker lapse rate was connected with the frontal layer and that the lower boundary was at 670 mb.

With constant inclination of the warm front layer the lower boundary should intersect the station vertical over Karlsborg at about the 550 mb level; the upper boundary should intersect the station vertical over Bromma about the 450 mb level. A layer over Karlsborg with marked weaker lapse rate from 580 mb and upward and an inversion between 450 and 483 mb over Bromma were therefore assumed to be connected with the frontal boundaries. Since the cyclone was formed on the Arctic front and an arctic air mass does not usually reach very high (normally it forms a dome), the warm front layer and the cold front layer were drawn together between Bromma and Jokioinen.

(b) *Isopleth diagrams*

The isopleth diagrams (Figs. 25 and 26) were constructed from serial wind measurements made by a radar set at Ursvik between 0915 and 1440 and from radio wind measurements from Bromma at 0635, 1235 and 1835. Since the wind measurements at 1200 and 1300 from Ursvik showed a very good agreement with the wind measurement from Bromma at 1235 and since the distance between the two places is only 3 km, all measurements were considered as made at Ursvik.

At the earth's surface the back-bent occlusion moved with an average speed of 40 km/h measured between 06^h and 09^h and in a direction along the cross-section (Fig. 21). With this speed value the distance scale of the cross-section was altered into a time scale on the assumption that "vicinity replaces succession". With this time scale the parts of the frontal boundaries falling between 06^h and 09^h were drawn in the isopleth diagrams. The back-bent occlusion layer could also be

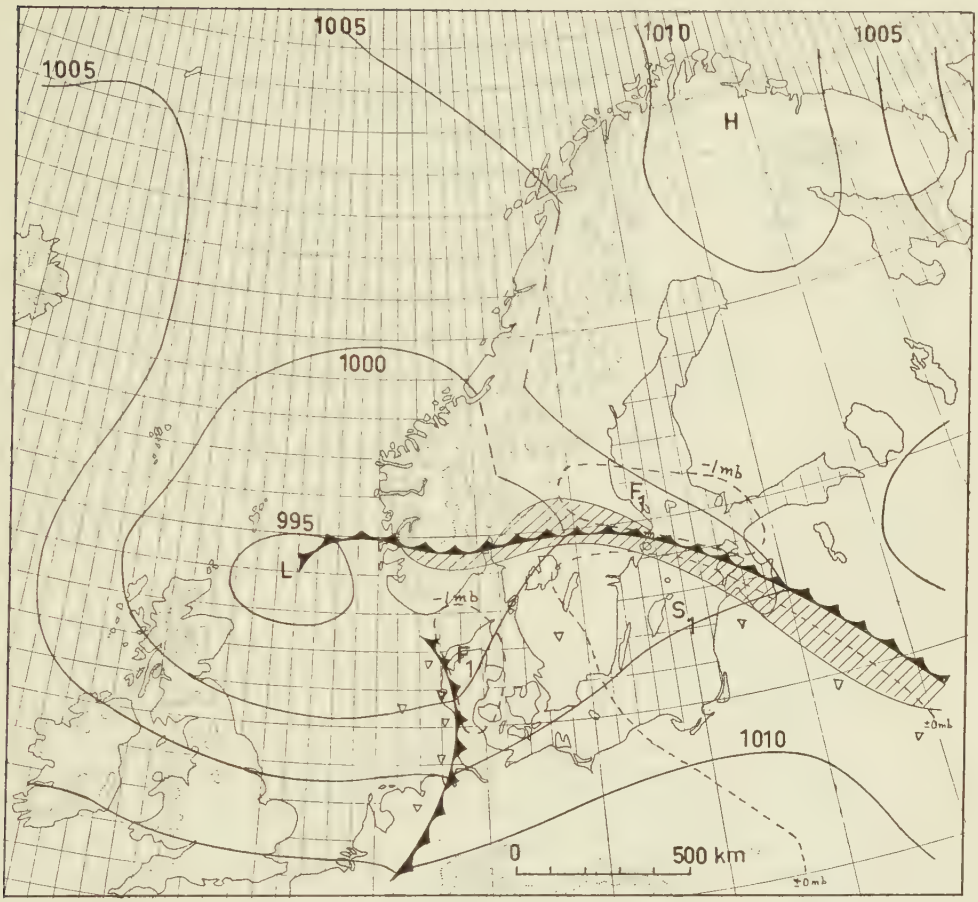

FIG. 27. Composite map, 0400 July 15, 1953.

settled in the diagrams by means of the tempogram from Bromma at 0635 (FF', Fig. 22).

The method of altering the distance scale into a time scale presupposes that the frontal system moved without any remarkable changes. This assumption is never exactly fulfilled, but in this case it seems probable that the structure of the front was not appreciably changed; the time interval between 06^h and 09^h is very short.

The warm front boundaries were settled in the isopleth diagrams by means of the tempograms from Bromma at 1235 and 1835 and the cross-section from Fig. 21.

Unfortunately, a good deal of the back-bent occlusion layer fell within the gap between the Bromma measurement of 0635 and the first measurement at Ursvik. Obtained wind values permitted, however, a rather safe interpolation of the isopleths within the gap.

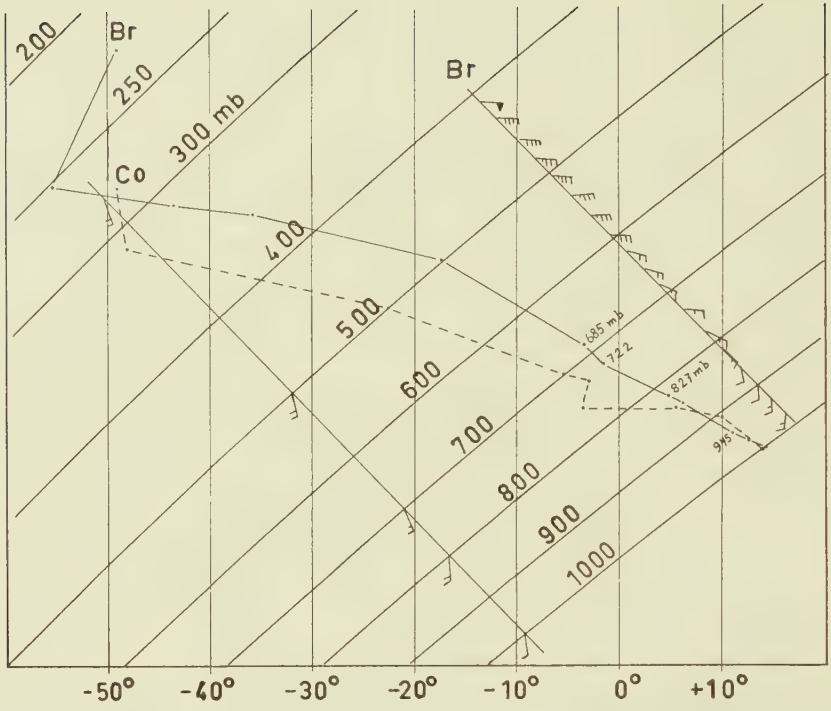

FIG. 28. Tempograms and winds from Bromma at 0415 (solid line) and Copenhagen (dashed line) at 0400 July 15, 1953. Observed winds are plotted in the conventional manner. Full barb 5 m/sec, half barb 2.5 m/sec, triangular barb 25 m/sec.

The isovel diagram shows the interesting fact that *a marked minimum in the wind speed was found within the frontal layers.* In the back-bent occlusion layer the minimum was connected with a rather sharp wind-turning, especially in the lower part of the layer. In the warm front layer, however, the wind turning was not at all so marked. The isogon diagram shows the well known wind-turning to the left with height above the frontal layers and a simultaneous increasing wind speed.

3.2 Example 2

Cold front.

Serial wind measurements from Ytterskär and Gåsvik, July 15, 1953, 09^h–17^h. (Briefly treated earlier by Eriksson, 1954.)

(a) *Synoptic situation*

During July 14 and the night of July 14–15, 1953 an old occlusion front belonging to a cylone over the North Sea moved in a NNE direction over the southern part of Sweden. This front had acquired the character of a cold front which is rather common

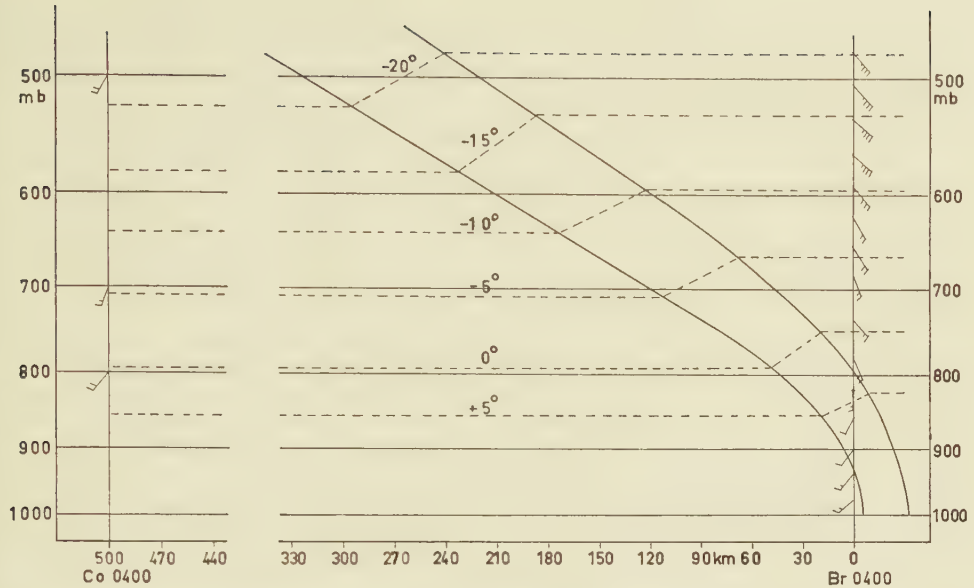

FIG. 29. Vertical cross-section, 0400 July 15, 1953 through Bromma and normal to the front at the earth's surface. Dashed lines are isotherms (°C), heavy lines are frontal boundaries. Observed winds are plotted in the conventional manner. Full barb 5 m/sec, half barb 2.5 m/sec.

in summer. At 0400 on July 15, according to the composite map (Fig. 27), the front extended in an easterly direction, from the south of Norway over Sweden to the Baltic Sea, where it turned to the SE and passed over the Balticum. The front appeared most distinctly in the wind field. Soutward from the front there was an area of rain. The average horizontal speed of the front in the direction NNE, i.e. normal to the frontal line at the earth's surface, was calculated at 30 km/h after having passed Ytterskär. This calculation was made from the composite maps plotted for every hour between 0800 and 1600.

No aerological soundings were available for constructing a vertical cross-section through the frontal layer at its most interesting portion, viz. a vertical section through Ytterskär and normal to the front at the earth's surface. The network of aerological stations in this region of interest, when the wind measurements were made, was too scanty.

With the aid of the Bromma sounding of 0415 and also the maps of 700 mb and 500 mb, the frontal boundaries, however, could be approximately determined in a vertical cross-section (Fig. 29). By constructing the cross-section the following considerations were made. Since the front at the earth's surface—accordong to the composite map at 0400—had just passed Bromma at that point of time, the frontal layer ought to be detectable in the lowest part of the Bromma tempogram of 0415 (Fig. 28).

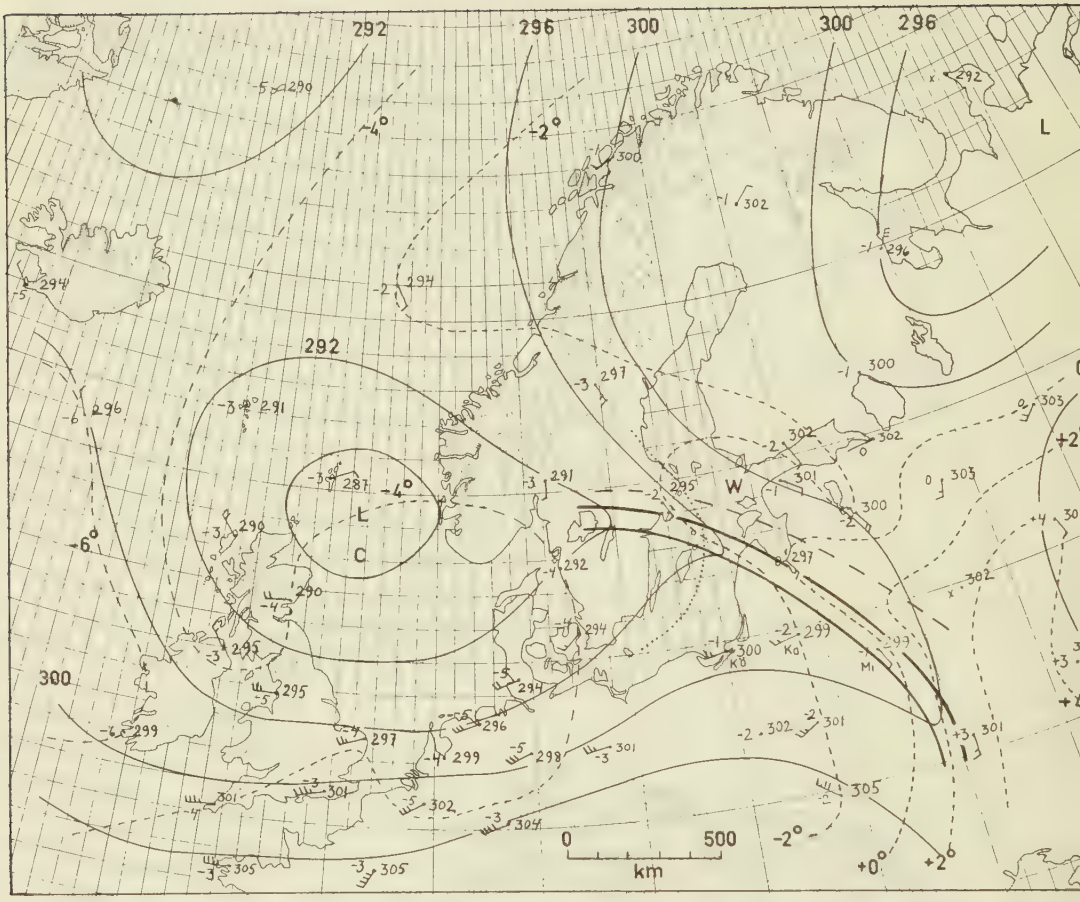

FIG. 30. 700 mb map, 0400 July 15, 1953. Solid lines are isohypses, drawn for every 4 geopotential decameter, dashed lines are isotherms for every 2°C, dotted line is an isohypse examplifying the normal way of isohypses in the vicinity of fronts, heavy lines are frontal boundaries. The heavy dashed line indicates the front at the earth's surface.

According to the marked wind-turning in that part of the tempogram a layer between 945 and 827 mb with somewhat weaker lapse rate was assumed to correspond to the frontal layer, but not necessarily to represent the frontal layer in its entirety. The layer between 722 and 685 mb with a more pronounced weaker lapse rate seems to have too little vertical thickness and to be at too high an altitude to correspond to the frontal layer. Therefore the lower boundary was estimated to intersect the station vertical over Bromma at 945 mb and the upper boundary—according to the wind values—at around 800 mb.

Both at the 700 mb and the 500 mb level (Figs. 30 and 31) the front was rather

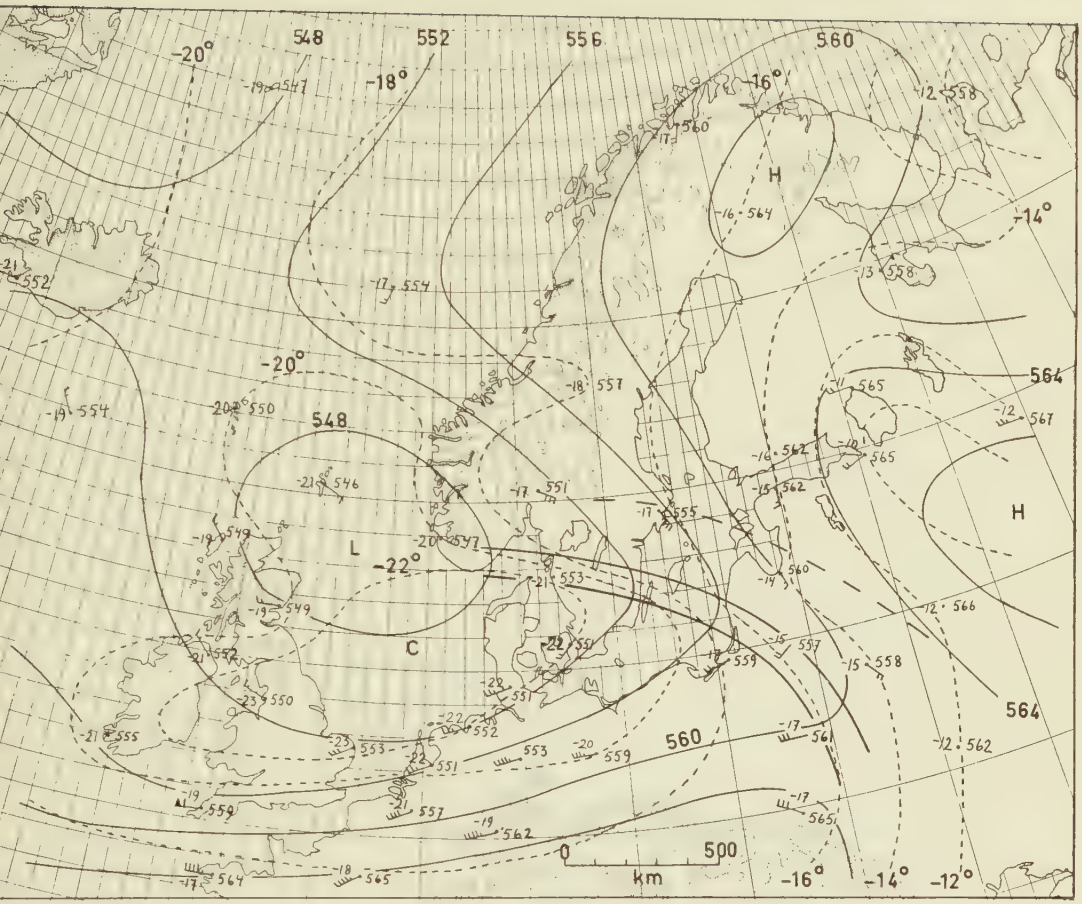

FIG. 31. 500 mb map, 0400 July 15, 1953. Solid lines are isohypses, drawn for every 4 geopotential decameter, dashed lines are isotherms for every 2°C, heavy lines are frontal boundaries. The heavy dashed line indicates the front at the earth's surface.

well indicated by a pronounced trough and also by the wind field, especially over the Balticum and western Russia. From this fact and with the aid of the composite map the frontal boundaries could be approximately settled at the levels mentioned and therefore also at the 700 mb and the 500 mb levels of the cross-section. The aerological ascent from Copenhagen at 0400 (Fig. 28) was chosen to represent the temperature conditions of the cold air mass behind the front.

With the average horizontal speed of 30 km/h for the front, the distance scale of Fig. 29 was altered into the time scale of the isopleth diagrams. The heights of the constant pressure levels were calculated, and so the frontal boundaries could be drawn in the isopleth diagrams. In this case the method of "vicinity replaces succes-

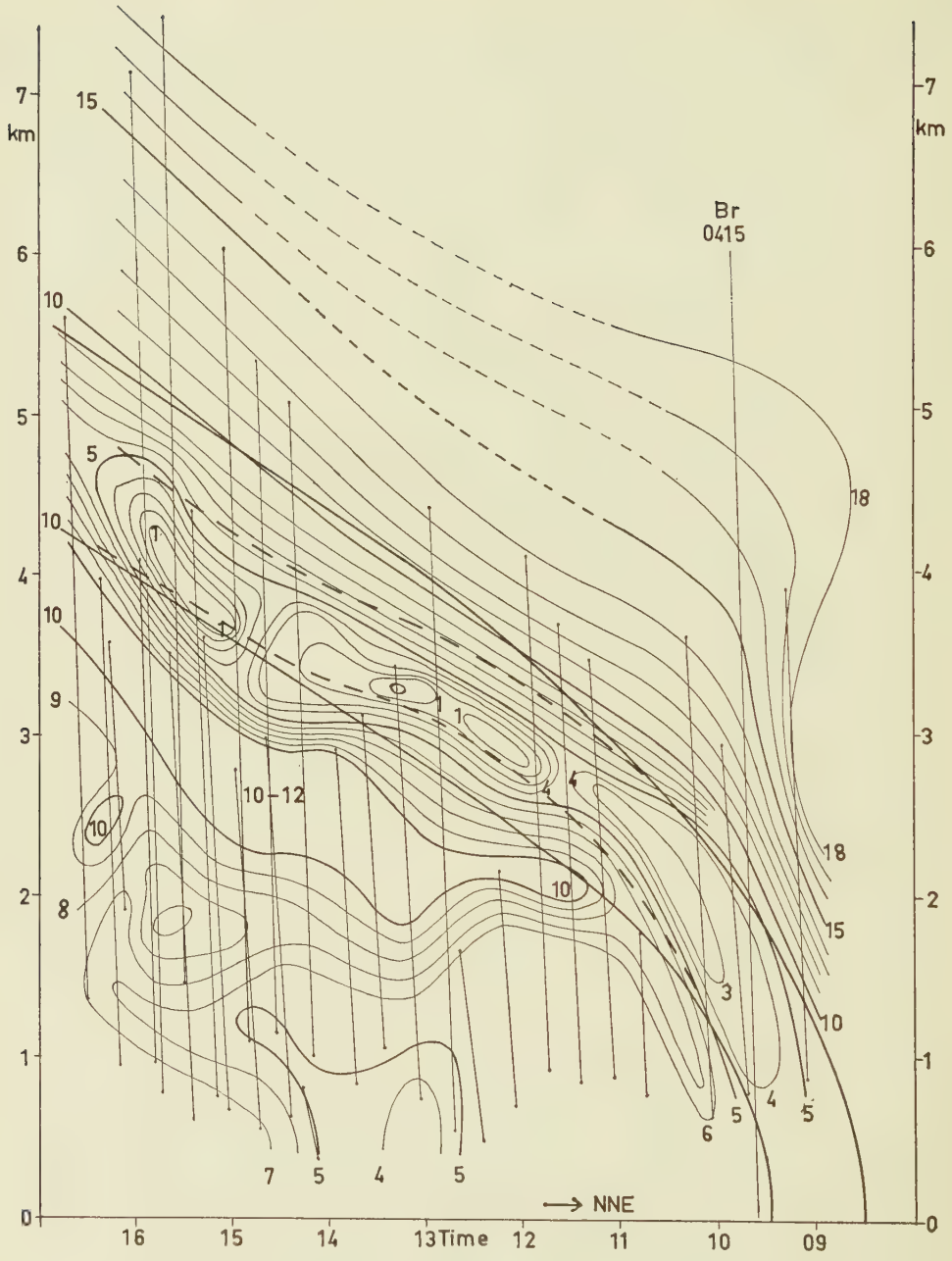

Fig. 32. Isovel diagram, Ytterskär and Gåsvik, July 15, 1953. Thin solid lines are isovels for every m/sec (*observed* wind), heavy lines are frontal boundaries obtained from the cross-section of Fig. 29 and heavy dashed lines modified frontal boundaries.

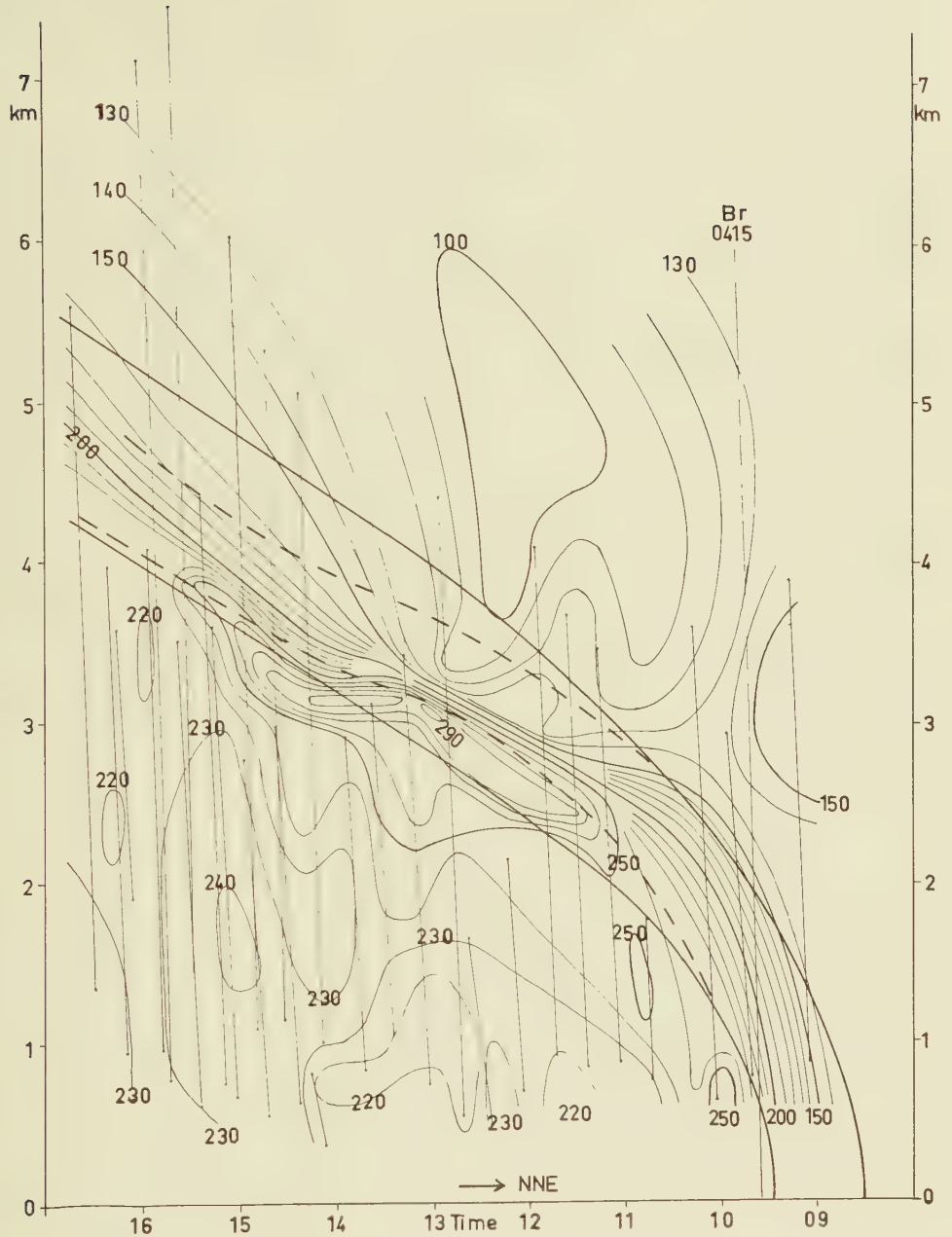

Fig. 33. Isogon diagram, Ytterskär and Gåsvik, July 15, 1953. Thin solid lines are isogons (degrees from the north) for *observed* wind, heavy lines are frontal boundaries obtained from the cross-section of Fig. 29 and heavy dashed lines modified frontal boundaries.

sion" seems to be rather well fulfilled, since from experience weak fronts of the present type have a tendency to keep their structure unchanged.

(b) *Isopleth diagrams*

The isopleth diagrams (Figs. 32 and 33) were constructed from wind values obtained from two independent radar sets, one at Ytterskär and the other at Gåsvik 5.6 km west of Ytterskär. The reason for this was that the measurements from Ytterskär only reached about 4 km and the measurements from Gåsvik, which reached 5–7 km, were only made during the latter part of the measuring period. The front did not pass the two places at the same time, but the time difference was calculated to be only four minutes. As the distance between the two places is only 5.6 km in a direction almost parallel to the front at the earth's surface, the wind values from Gåsvik were plotted together with the values from Ytterskär in the same isopleth diagram but with a displacement of four minutes relatively to the values from Ytterskär. Those wind values, which were made simultaneously, showed a very good agreement. In order to complete the isopleth diagrams the Bromma ascent of 0415 was fitted to the diagrams with the help of the wind values.

The wind discontinuity appears surprisingly distinct both in the isogon and the isovel field. The wind turns 50–100° with an altitude difference of only a few hundred metres. Especially in the layer around 3 km there is a vertical change of about 130° in only 150 m. Also in this example there is *a well marked minimum in the wind speed within the frontal layer*. In this case, however, the speed minimum is coupled to a

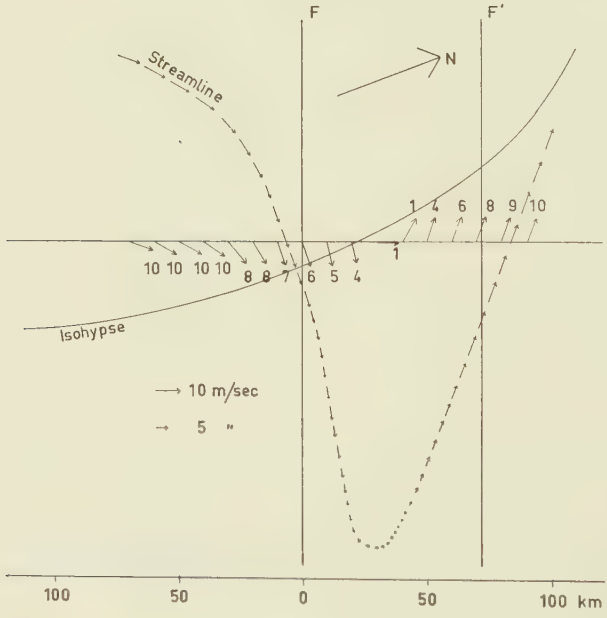

Fig. 34. Streamline and isohypse at the 700 mb level (3 km), July 15, 1953.

much more pronounced wind-turning than in the previous example (January 19, 1960). The characteristic increase in wind speed above the frontal layer and the simultaneous wind turning to the left with height are very distinct.

It is probable from the isopleth diagrams that the frontal layer indicated in the vertical cross-section in Fig. 29 has been given a somewhat too great vertical thickness. It also seems as if the slope should have been different, approximately as given by the heavy dashed lines drawn in the isopleth diagrams. Thus constructed, the slope of the frontal layer is not regular, but such a slope *may* occur (see, for instance, Bjerknes & Palmén, 1933).

It is a well-known fact that in the peripheral parts of a cyclone the streamlines behind a cold front—corresponding to the greater slope of the frontal surface in lower levels—have an anticyclonic curvature immediately behind the front near the earth's surface. At higher altitudes, however, a cyclonic curvature of the streamlines is assumed. This is clear from the normal way of drawing isohypses in the vicinity of fronts. However, the isogon diagram shows the interesting fact that *even at higher levels, in this example around the 700 mb level (3 km), there may occur an anticyclonic curvature of the streamlines behind the front although the slope of the frontal layer* is not very steep.

The anticyclonic curvature is more clearly seen from Fig. 34, which shows a detail of the wind field at the 700 mb level (3 km). Fig. 34 is constructed from the isopleth diagrams by altering the time scale into a distance scale, using as above the value of 30 km/h for the average horizontal speed of the front. Thus wind values could be obtained in certain horizontal distances from the front. In Fig. 34 the two parallel straight lines F and F' indicate the front at the 3 km level, where F' is the boundary towards the warmer side (F and F' refer to the frontal layer as it was obtained from the vertical cross-section of Fig. 29). The arrows on the straight line normal to the front indicate the wind direction at certain distances on both sides of the front. The figures at the arrows give the wind speed in m/sec. Both direction and speed were obtained from the isopleth diagrams. From the direction arrows a streamline was constructed. It is obvious from Fig. 34 that the streamlines at the 700 mb level have, in this case, a pronounced anticyclonic curvature behind the front. Therefore the isohypses on the 700 mb map in Fig. 30 were also drawn with an anticyclonic curvature behind the front. This way of drawing is moreover confirmed by the gpm-values given by Königsberg (Kaliningrad), Kaunas and Minsk stations (Kö, Ka and Mi, Fig. 30). The normal way of drawing the isohypses in this case is illustrated by the isohypse in Fig. 34, which is obtained from the upper-air analysis on a working 700 mb map of 0400 on the actual day and also indicated by the dotted isohypse of Fig. 30.

The anticyclonic curvature of the streamlines behind the front at the 700 mb level is not just a peculiarity of the front in this example. It also occurs in the following example. The examples were not specially selected but were obtained from wind measurements which happened to be made in connection with the passage of a front.

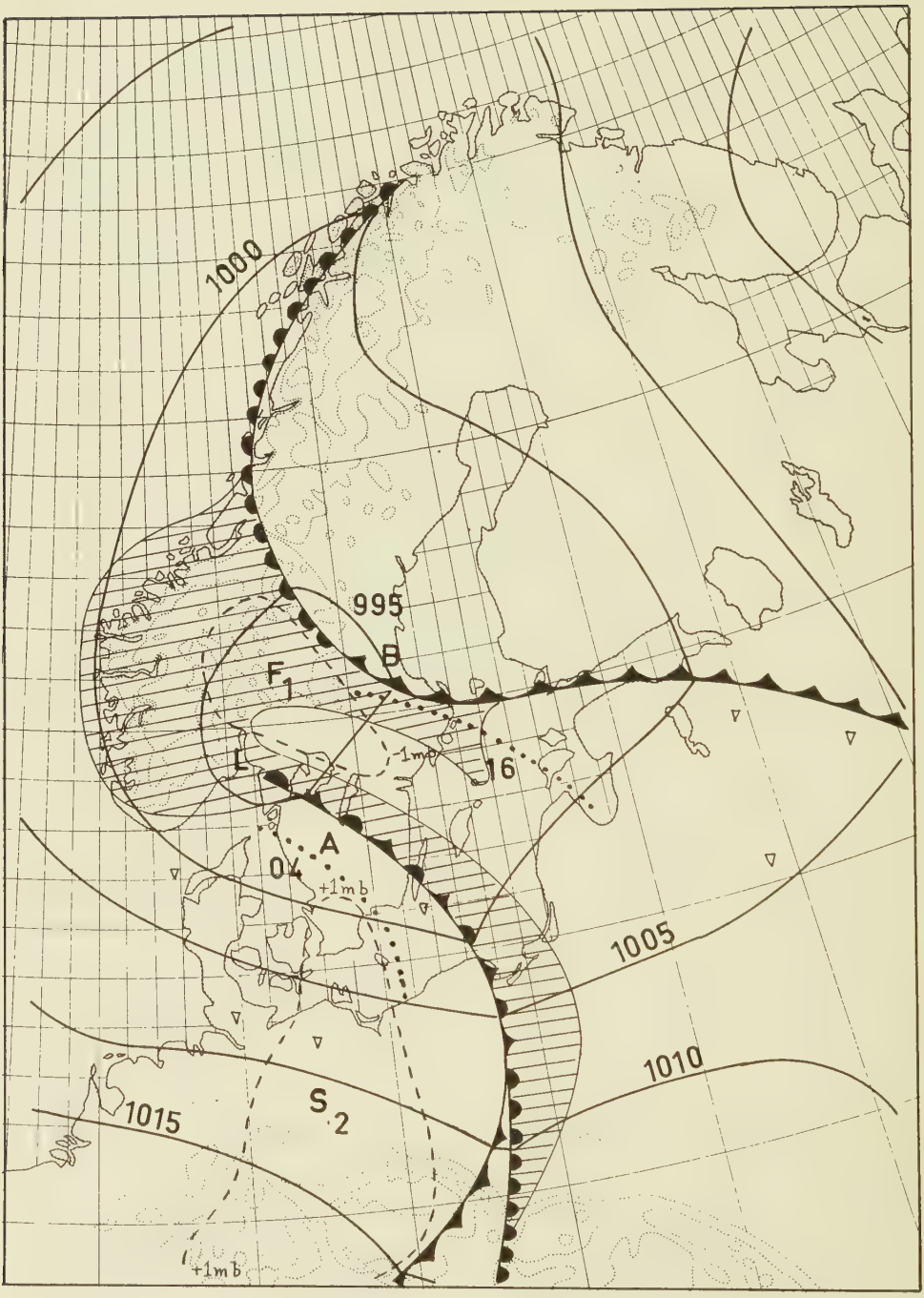

FIG. 35. Composite map, 0700 July 15, 1954.

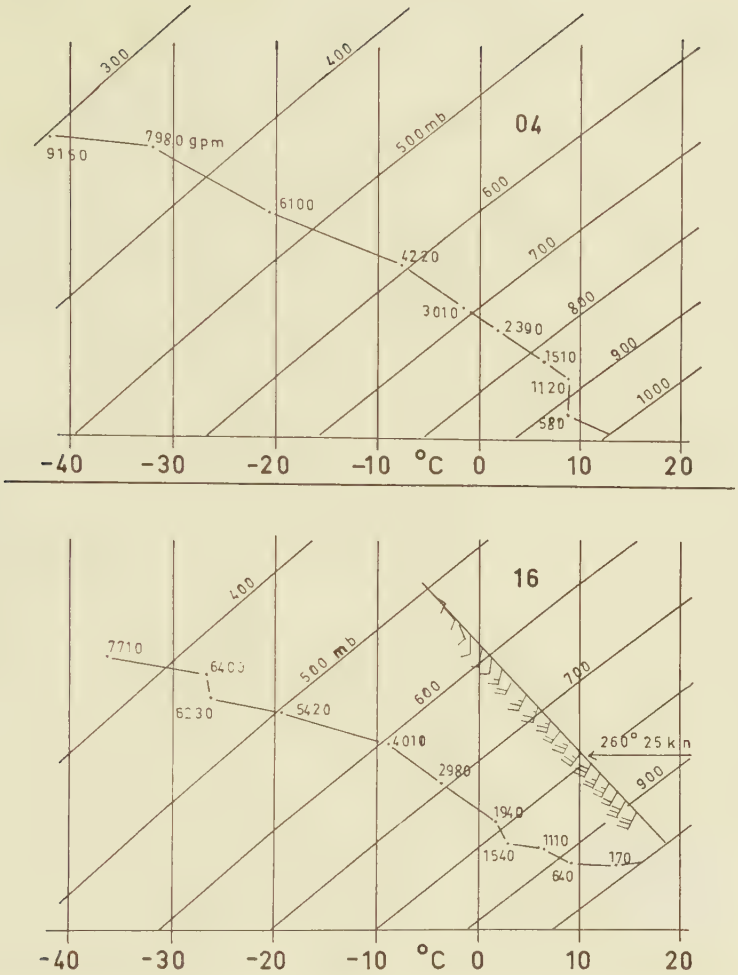

Fig. 36. Tempograms from Torslanda at 0400 and 1600 July 15, 1954. Observed winds are plotted in the conventional manner. Full barb 5 m/sec, half barb 2.5 m/sec.

3.3 Example 3

Occlusion front.

Serial wind measurements from Gothenburg, July 15, 1954, 09^h–16^h.

(a) *Synoptic situation*

On this day an occlusion front (front *A* of Fig. 35) belonging to a cyclone over the middle part of Scandinavia moved in a NE direction over the southern part of Sweden. According to the composite map of 0700 (Fig. 35), the front extended over Götaland

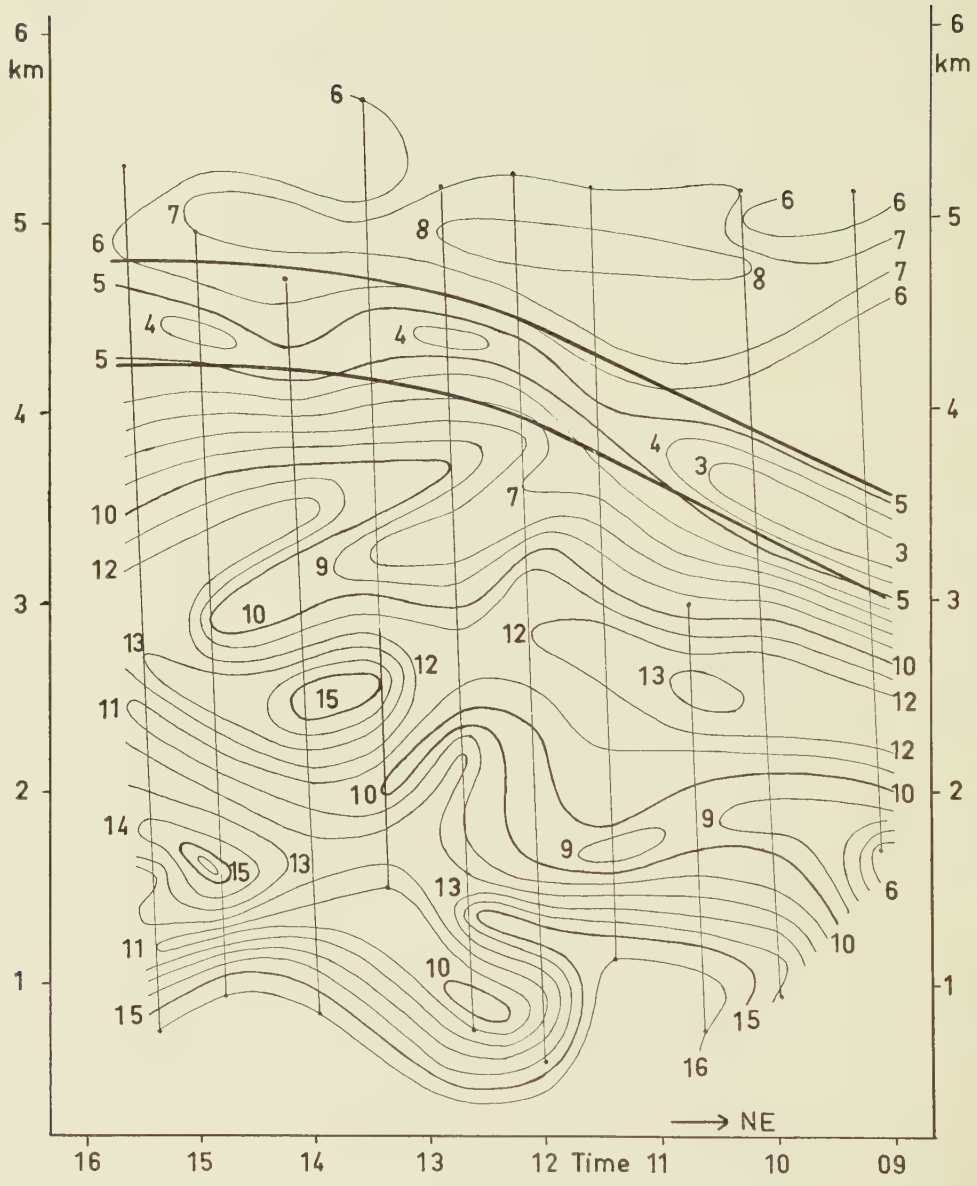

Fig. 37. Isovel diagram, Gothenburg, July 15, 1954. Thin solid lines are isovels for every m/sec (*observed* wind), heavy lines are frontal boundaries.

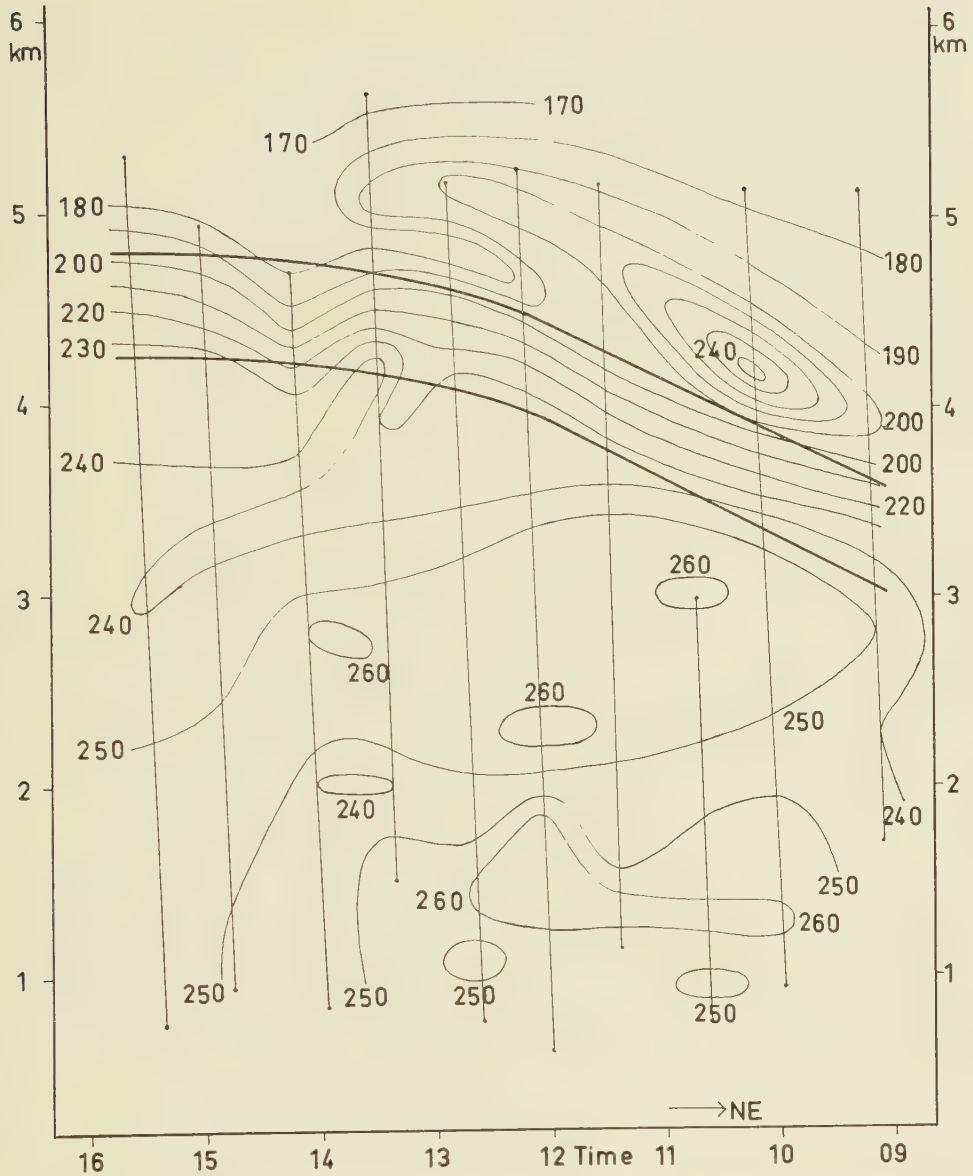

FIG. 38. Isogon diagram, Gothenburg, July 15, 1954. Thin solid lines are isogons (degrees from the north) for *observed* wind, heavy lines are frontal boundaries.

and the Baltic Sea to Germany. The position of the front at 0400 and 1600 is marked on the map in Fig. 35 by the two dotted lines. North-east of this front, another front (front *B*) at 0700 extended from the Baltic over Åland to middle of Sweden and from there to the north over the Scandinavian highland. In its easterly portions this front had the character of a cold front, over Scandinavia of a warm front.

In the tempograms from Torslanda (Fig. 36) the layer of front *A* was rather well indicated at 0400 by the isothermal layer between 580 and 1120 gpm. At 1600 the settlement of the frontal boundaries was more doubtful. The lower boundary was, however, settled by means of the wind values and the nearly isothermal layer between 6230 and 6400 gpm was assumed to be connected with the upper boundary of the frontal layer.

From the facts given above the probable location of the frontal boundaries was indicated by two heavy lines in the isopleth diagrams.

(b) *Isopleth diagrams*

The isopleth diagrams (Figs. 37 and 38) were constructed from wind values obtained from a radar set at Gothenburg. Also in this case there was a *marked minimum in the wind speed within the frontal layer*. Immediately above the frontal layer the wind speed as usual increased with height and simultaneously the wind turned to the left. However, between 10^h and 13^h this normal turning of the wind ceased within a few hundred metres above the upper frontal boundary. Here the wind instead turned to the right within a layer having a thickness of some few hundred metres. Perhaps this turning to the right and this decreasing wind speed above the level of 5 km may indicate that there may have been another frontal layer above the one indicated in the diagrams, perhaps front *B*.

In this case also, it is seen from the isogon diagram that *there is an anticylonic turning of the wind behind the front at a level around 3 km*. Since the frontal layer during the measuring period was situated above this latter level, an intersection between the frontal layer and the 3 km level was not obtainable with a reasonable degree of accuracy by extrapolating the isogons. Therefore no diagram of the wind field of the same type as in Fig. 34 has been constructed.

3.4 Example 4

Inversion layers.

Serial wind measurements from Ytterskär, May 25, 1954, 08^h–16^h.

(a) *Synoptic situation*

According to the composite map at 1300 (Fig. 39), a ridge of high pressure was situated over Scandinavia. The ridge was found at all levels up to and above the 225 mb level; it had existed for several days. The aerological ascents from Bromma

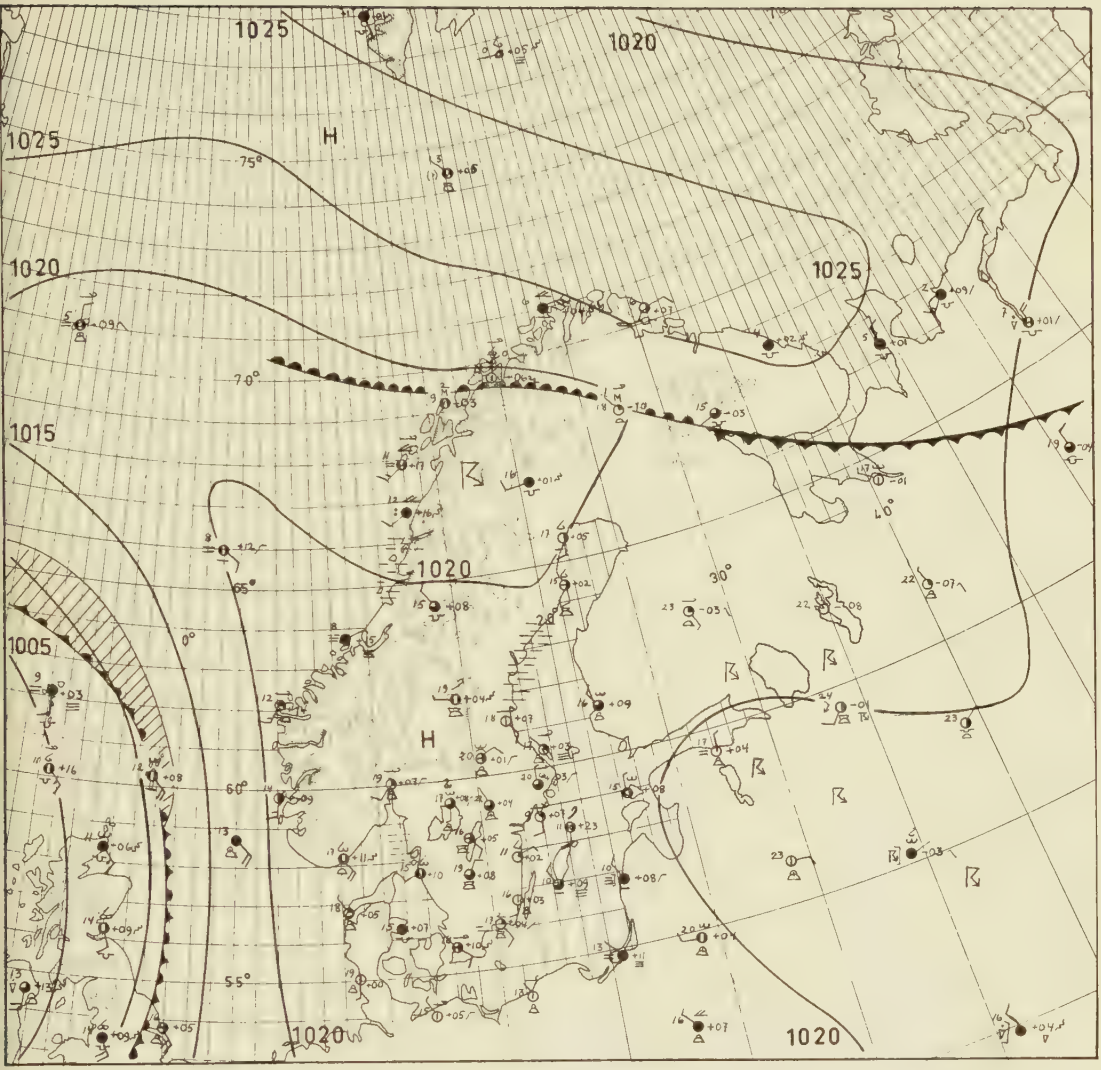

Fig. 39. Composite map, 1300 May 25, 1954.

(Fig. 40) showed both at 0400 and 1600 several layers with inversion or with weak lapse rates, undoubtedly caused by subsidence. Layers *C* and *D* disappeared during the day and probably layers A and B descended to positions *A'* and *B'*. A new layer *E* with a weak lapse rate was created. The wind speed was rather low and the direction changed with height, apparently in a very irregular way. The wind was, as people say, "light and variable".

The aerological ascent from Bromma at 0400 showed that the wind over layer *A*

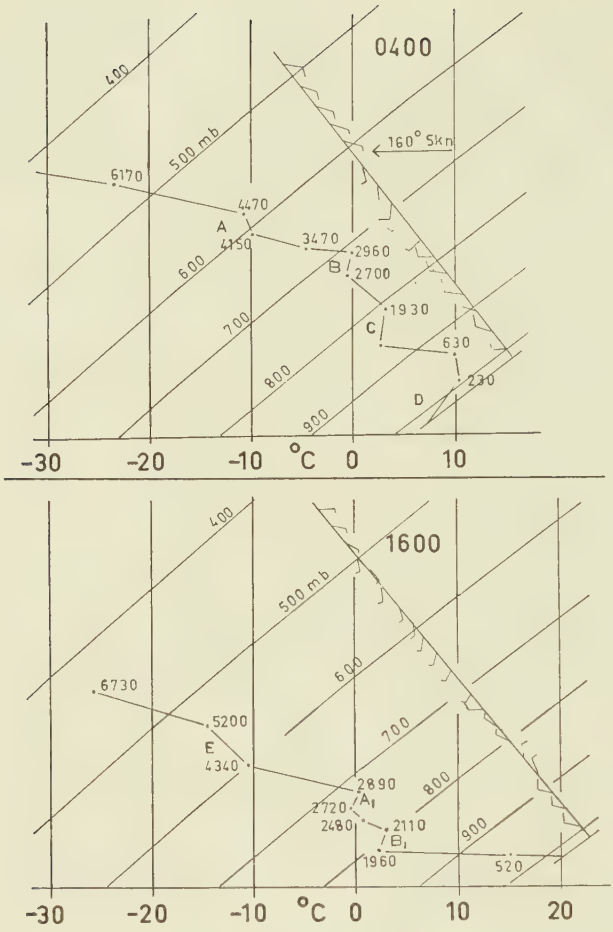

FIG. 40. Tempograms and winds from Bromma at 0400 and 1600 May 25, 1954. Observed winds are plotted in the conventional manner. Full barb 5 m/sec, half barb 2.5 m/sec.

in Fig. 40 was from about 160° and somewhat increasing with height. In this synoptic situation one might assume that layer *A* would descend during the day and therefore also the layer with wind direction 160°. This in fact happened to begin with, but then the wind turned to 260°–300° (Fig. 41). The wind direction 160° could now be found at altitudes about 1 km higher up than in the morning. An explanation of this will be given below in connection with our discussion of the isopleth diagrams.

(b) *Isopleth diagrams*

The isopleth diagrams are shown in Figs. 41 and 42. The isogons show very distinct discontinuities, manifestly in connection with layers *A'*, *B'*, *C* and *E*. It is obvious that the wind turned with height in an anticyclonic way beneath the inversion layers and in a cyclonic way above them. This is the reason for the unexpected wind-

turning from 160° to 300° as mentioned above. It was caused by the creation of layer *E*. Also, a wind speed minimum is connected with the inversion layers.

Example 4 shows the importance of taking the effect of temperature inversions into consideration when giving upper-wind data in reports as well as in forecasts. When the wind speed is low, the behaviour of eventual inversions may have a great influence upon the wind direction. In spite of the low wind speed this is of importance, for instance, in air pollution problems. Knowing the effect of inversion layers upon wind distribution, it is thus not necessary to use expressions like "light and variable wind" which are really of not much help in practice.

4. Conclusions

The examples given above were not taken from selected situations. It was a mere chance that the serial wind measurements were made, or in Example 1 could be made in connection with the passage of a front. Of course the examples are rather few, but in spite of this they ought to be rather good random samples. The following conclusions may therefore have a general bearing.

The wind-speed minimum, established within the frontal layers in all the examples given here, seems to be a *characteristic feature of frontal layers* in the troposphere. Example 2 shows that the wind minimum is well developed, even if the frontal layer appears very indistinctly in the tempograms. Wind measurements may therefore be a more useful tool than radio-soundings for the settlement of frontal layers.

The wind minimum within a frontal layer which we have found empirically does not correspond with the usual method of drawing the isovels in connection with the tropospherical part of a frontal layer. In several investigations of the distribution of upper winds (for instance, Palmén, 1948*b*; Palmén & Nagler, 1948; Palmén & Newton, 1948; Hovmöller, 1948; Berggren, 1953) there are cross-sections containing frontal layers showing both specified characteristic examples and average conditions. In all cases, however, the isovels are drawn in a way indicating a continuous increase of the wind speed with height within the frontal layer. The isovels were, however, in all these cases constructed on the basis of geostrophic wind values or on observed winds (synoptic upper-wind observations) much less detailed than in the present study. Therefore these wind values did not permit any detailed isovel analysis in connection with fronts, nor was such an analysis the aims of these investigators. Therefore the cross-sections given in these previous investigations may only disagree with ours because of their lack of detailed observations.

Obviously there may occur an anticyclonic curvature of the streamlines behind a cold front also in the free atmosphere. In our examples it occurs around the 700 mb level. This is of a great practical importance concerning the determination of the upper-wind direction from constant pressure maps. Using contour lines—as they are normally drawn—for estimating wind direction immediately behind cold fronts,

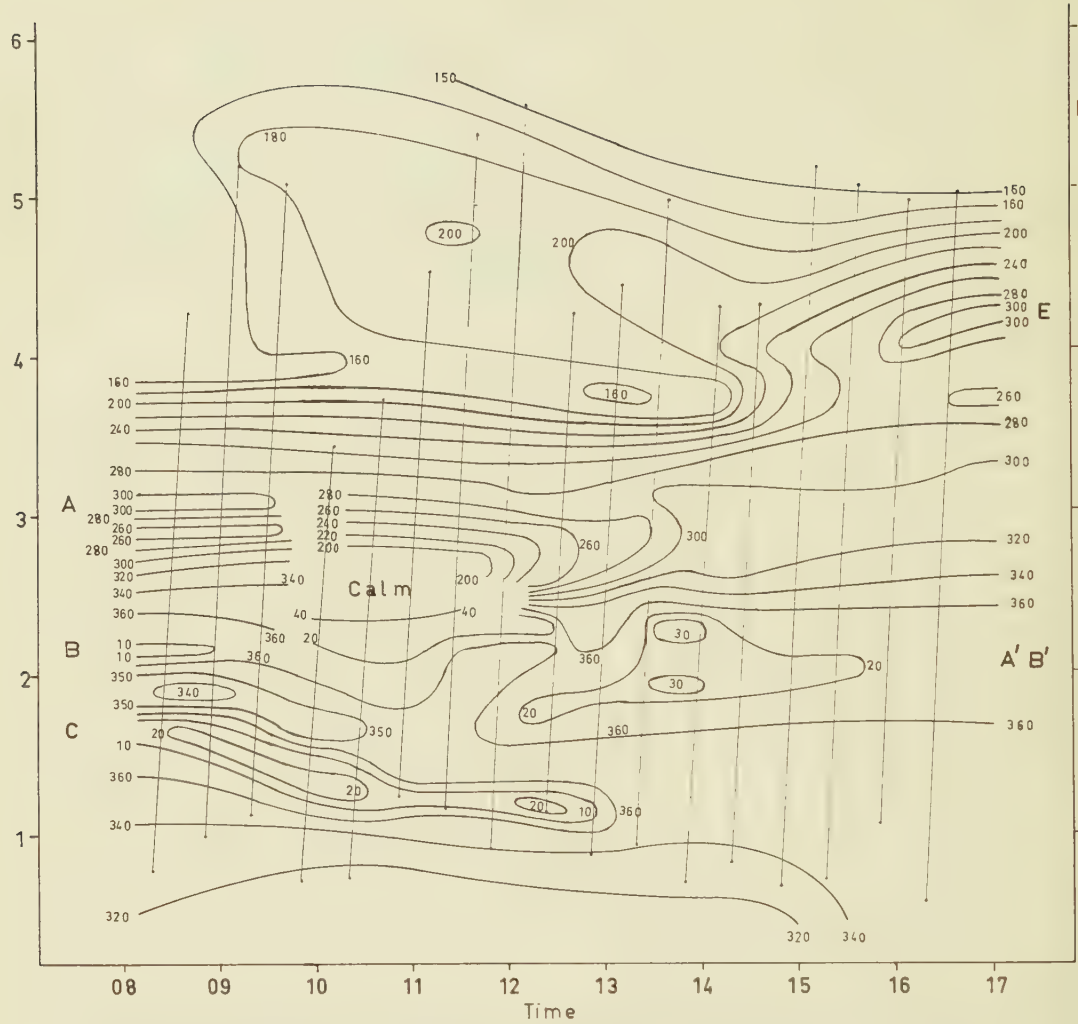

FIG. 41. Isogon diagram, Ytterskär, May 25, 1954. Solid lines are isogons (degrees from the north) for *observed* wind.

there may be an appreciable difference between the data obtained and the real wind values. In our example (Example 2, Fig. 34), this difference is 30°–90° in a zone some 100 km wide. When such a difference occurs in connection with wind speeds of 4–10 m/sec, as in our example, it may be of a considerable practical importance. It seems that one must pay more consideration to the gpm-values on the upper-air maps than is usually the case when analysing these maps. An anticyclonic wind-turning behind a cold front may occur and the wind-turning may also take place in a rather abrupt way, at upper levels, not only at the earth's surface.

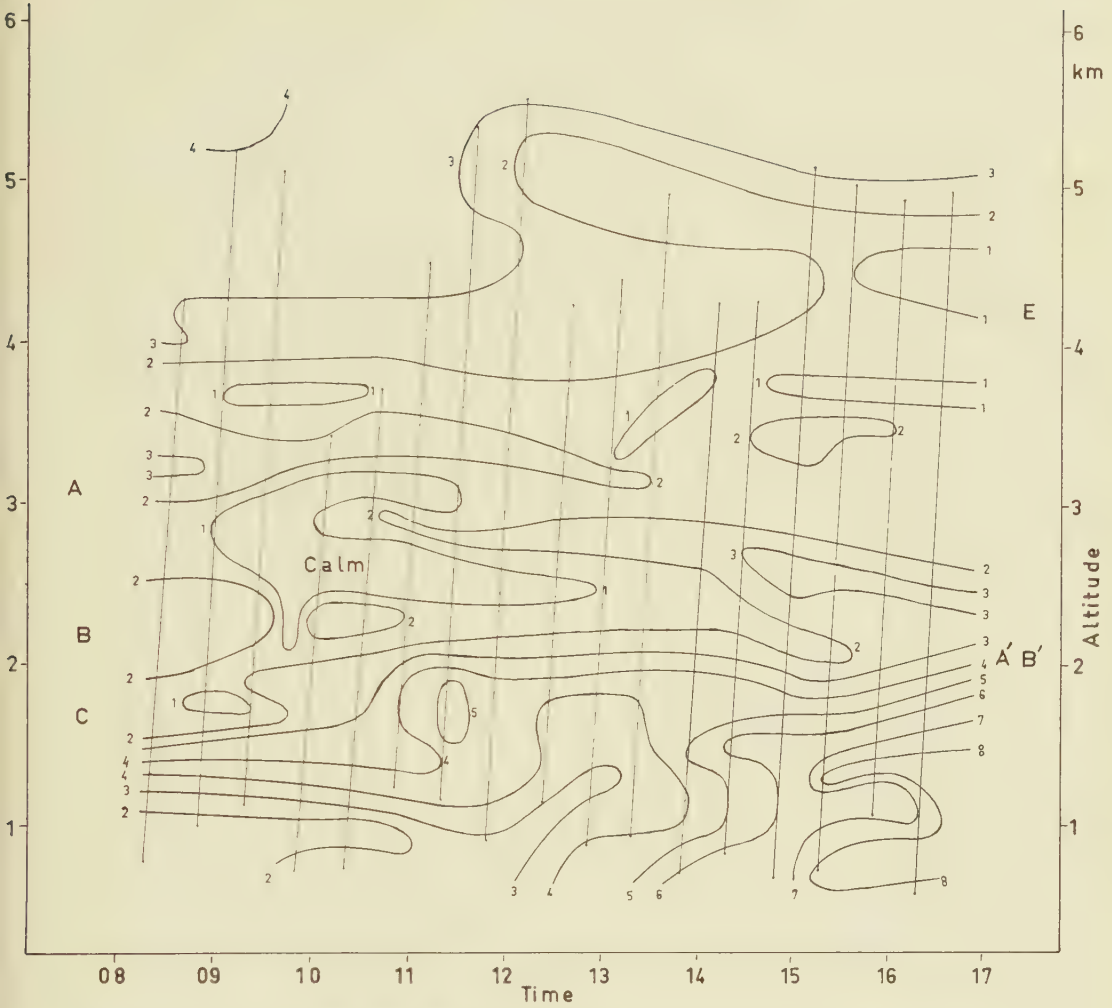

Fig. 42. Isovel diagram, Ytterskär, May 25, 1954. Solid lines are isovels for every m/sec (*observed* wind).

The anticyclonic curvature of the streamlines behind a front is obviously connected with the field of vertical movement and divergence in the lower parts of the cold air mass just behind the front. *Therefore the contour lines on upper-air maps ought to be given an anticyclonic curvature behind a cold front in all levels in the lower troposphere when indicated by wind and gpm-values.* In the upper part of the tropopshere and in the lower stratosphere, however, cyclonic curvature should prevail, corresponding to general convergence above the sinking tropospheric air masses.

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